CHAPTER-4

MODELING OF FLUID FLOW IN HYDRAULIC FRACTURES

4.1 Material Balance Equation for flow of gas in Hydraulic Fracture:

4.1.1 Assumptions:

1) No mass transfer between the fluids.
2) Average width of the fracture is $\overline{w_f}$.
3) The entire flow process is isothermal. (T= Constant).
4) Hydraulic fracture is perpendicular to the horizontal well bore.
5) The geometry of the hydraulic fracture is rectangle.

The schematic representation of gas flow in the hydraulic fracture is shown in Figure 4.1

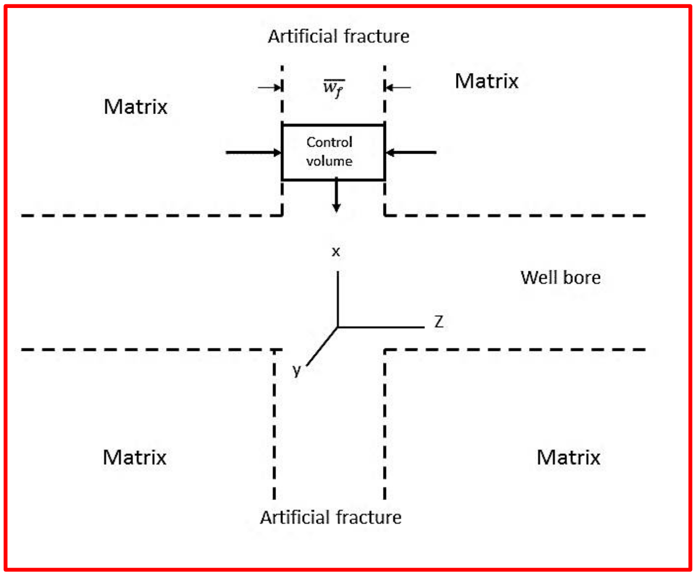


Figure 4.1: Schematic flow of gas in the hydraulic fracture.

Material Balance Equation for the gas flow through the hydraulic fracture:

$(Accumulation\ with\ in\ system)$=$(Gas\ flow\ in\ through\ system\ boundaries)$-$(Gas\ flow\ out\ through\ system\ boundaries)$+$(Generation\ with\ in\ system)$-$(Consumption\ within\ system)$.

As control volume decreases, the volume = Δx.Δy.$\overline{w_f}$.

$$\Rightarrow -\left(v_{g_{fx}}\rho_{gf}|_{x+\Delta x}\Delta y.\overline{w_f}\right)-\left(-v_{g_{fx}}\rho_{gf}|_{x}\Delta y.\overline{w_f}\right)+\left(-v_{g_{fy}}\rho_{gf}|_{y+\Delta y}\Delta x.\overline{w_f}\right)-\left(-v_{g_{fy}}\rho_{gf}|_{y}\Delta x.\overline{w_f}\right)+\left(-v_{g_{mz}}\rho_{gm}|_{z=\frac{-\overline{w_f}}{2}}\Delta x\Delta y\right)-\left(-v_{g_{mz}}\rho_{gm}|_{z=\frac{\overline{w_f}}{2}}\Delta x\Delta y\right)= \frac{\Delta\left((\Delta x\Delta y\overline{w_f}.\emptyset_f).\rho_{gf}.S_{gf}\right)}{\Delta t} \text{-------}\rightarrow \mathbf{1.}$$

Dividing equation 1 by Δx.Δy.$\overline{w_f}$, We got

$$\Rightarrow \frac{\left(-v_{g_{fx}}\rho_{gf}|_{x+\Delta x}+v_{g_{fx}}\rho_{gf}|_{x}\right)}{\Delta x}+\frac{\left(-v_{g_{fy}}\rho_{gf}|_{y+\Delta y}+v_{g_{fy}}\rho_{gf}|_{y}\right)}{\Delta y} + \frac{\left(v_{g_{mz}}\rho_g|_{z=\frac{-\overline{w_f}}{2}}-v_{g_{mz}}\rho_g|_{z=\frac{\overline{w_f}}{2}}\right)}{\left(\frac{-\overline{w_f}}{2}\right)} = \frac{\Delta\left((\emptyset_f).\rho_{gf}.S_{gf}\right)}{\Delta t} \text{-------}\rightarrow \mathbf{2.}$$

Taking limit Δx , Δy, Δz and Δt ---→ 0,

$$\Rightarrow -\frac{\partial\left(v_{g_{fx}}\rho_{gf}\right)}{\partial x} - \frac{\partial\left(v_{g_{fy}}\rho_{gf}\right)}{\partial y} - \frac{\left(v_{g_{mz}}\rho_{gm}\right)}{\frac{-\overline{w_f}}{2}} = \frac{\partial\left((\emptyset_f).\rho_{gf}.S_{gf}\right)}{\partial t} \text{-------}\rightarrow \mathbf{3.}$$

Considering Darcy's law, We have

$$\Rightarrow v_{g_{fx}} = \frac{-k_{gf}}{\mu_{gf}}\frac{\partial(P_{gf})}{\partial x} \text{-------}\rightarrow \text{for x-direction}$$

But, as the flow in the hydraulic fracture is two phase flow, the concept of relative permeability comes into act.

Relative permeability of gas with respect to water (k_{rgf})

$$\Rightarrow k_{rgf} = \frac{k_{gf}}{k_f} \text{ and}$$

$$\frac{k_{gf}}{k_f} \leq 1.$$

Now, $k_{gf} = k_{rgf}\ k_f$

Where, k_f= Absolute Permeability in fracture with proppant.[m^2].

k_{gf} = Effective permeability of gas in the fracture.

k_{rgf} = Relative permeability of gas in the fracture.

Now,

⇨ $-\upsilon_{g_{fx}} = \frac{-k_{rgf}\, k_f}{\mu_{gf}} \frac{\partial(P_{gf})}{\partial x}$ ----------------→ for x-direction.

⇨ $-\upsilon_{g_{fy}} = \frac{-k_{rgf}\ k_f}{\mu_{gf}} \frac{\partial(P_{gf})}{\partial y}$ ----------------→ for y-direction.

⇨ $-\upsilon_{g_{mz}} = \frac{-k_{gm}}{\mu_{gm}} \frac{\partial(P_{gm})}{\partial z}$ ----------------→ for z-direction.

Substituting $\upsilon_{g_{fx}}$, $\upsilon_{g_{fy}}$, $\upsilon_{g_{mz}}$ in equation 3, we get

⇨ $-\frac{\partial\left(\frac{-\beta_c\, k_{rgf}\ k}{\mu_{gf}} \frac{\partial(P_{gf})}{\partial x}\rho_{gf}\right)}{\partial x} - \frac{\partial\left(\frac{-\beta_c\, k_{rgf}\ k}{\mu_{gf}} \frac{\partial(P_{gf})}{\partial y}\rho_{gf}\right)}{\partial y} - \frac{\left(\frac{-\beta_c k_{gm}}{\mu_{gm}} \frac{\partial(P_{gm})}{\partial z}\rho_{gm}\right)}{\frac{-w_f}{2}} = \frac{\partial\left((\emptyset_f).\rho_{gf}.S_{gf}\right)}{\partial t}$ -→**4.**

$Where,$

β_c= Transmissibility conversion factor = $1.127\frac{scf}{D-psi}$.

$From\ equation\ of\ state,$

Formation Volume Factor ((B_g) = $\frac{\rho_{gsc}}{\alpha_c \rho_{gf}}$

From equation 4,

⇨ $\frac{\partial\left(\frac{\beta_c\, k_{rgf}\ k}{\mu_{gf}} \frac{\partial(P_{gf})}{\partial x} \frac{\rho_{gsc}}{\alpha_c B_g}\right)}{\partial x} + \frac{\partial\left(\frac{\beta_c\, k_{rgf}\ k}{\mu_{gf}} \frac{\partial(P_{gf})}{\partial y} \frac{\rho_{gsc}}{\alpha_c B_g}\right)}{\partial y} + \frac{\left(\frac{\beta_c k_{gm}}{\mu_{gm}} \frac{\partial(P_{gm})}{\partial z} \frac{\rho_{gsc}}{\alpha_c B_g}\right)}{\frac{-w_f}{2}} = \frac{\partial\left((\emptyset_f).\frac{\rho_{gsc}}{\alpha_c B_g}.S_{gf}\right)}{\partial t}$ -→ **5.**

From literature, $\emptyset_f = \emptyset_o e^{c_f\,(P_f - P_o\,)}$.

$\frac{\partial \emptyset_f}{\partial p_f} = \emptyset_f c_f$.

$Now, \frac{\partial \emptyset_f}{\partial t} = \frac{\partial \emptyset_f}{\partial p_f} \frac{\partial p_f}{\partial t}$

⇨ $\frac{\partial \emptyset_f}{\partial t} = \emptyset_f c_f \frac{\partial p_f}{\partial t}$.

Equation 5 can be rewritten as

$$\Rightarrow \frac{\partial\left(\frac{\beta_c k_{rgf}\ k}{\mu_{gf}}\frac{\partial(P_{gf})}{\partial x}\frac{\rho_{gsc}}{\alpha_c B_g}\right)}{\partial x} + \frac{\partial\left(\frac{\beta_c k_{rgf}\ k}{\mu_{gf}}\frac{\partial(P_{gf})}{\partial y}\frac{\rho_{gsc}}{\alpha_c B_g}\right)}{\partial y} + \frac{\left(\frac{\beta_c k_{gm}}{\mu_{gm}}\frac{\partial(P_{gm})}{\partial z}\frac{\rho_{gsc}}{\alpha_c B_g}\right)}{\frac{-w_f}{2}} = \left[\left(\frac{\rho_{gsc}}{\alpha_c B_g}\right) S_{gf}\emptyset_f c_f\right]\frac{\partial p_f}{\partial t} \longrightarrow \mathbf{6.}$$

Dividing equation 6 with $\Delta x\Delta y\Delta z$, we get

$$\Rightarrow \frac{\partial\left(\frac{\beta_c k_{rgf}\ k}{\mu_{gf}}A_x\frac{\partial(P_{gf})}{\partial x}\frac{\rho_{gsc}}{\alpha_c B_g}\right)}{\partial x}\Delta x + \frac{\partial\left(\frac{\beta_c k_{rgf}\ k}{\mu_{gf}}A_y\frac{\partial(P_{gf})}{\partial y}\frac{\rho_{gsc}}{\alpha_c B_g}\right)}{\partial y}\Delta y + \frac{\left(\frac{\beta_c k_{gm}}{\mu_{gm}}\Delta X\Delta Y\frac{\partial(P_{gm})}{\partial z}\frac{\rho_{gsc}}{\alpha_c B_g}\right)}{\frac{-1}{2}} = \left[\left(\frac{V_b\rho_{gsc}}{\alpha_c B_g}\right)(1-S_{wf})\emptyset_f c_f\right]\frac{\partial p_f}{\partial t} \longrightarrow \mathbf{7.}$$

In this case, the flow is a multiphase flow i.e. gas and water will flow through the fracture. So, the saturation values of gas and water will change with respect to time during production. Initially, water saturation will be nearly 100% and then the water saturation decreases and the gas saturation increases. A well bore has been considered in the 5th layer from the top of the reservoir. As the hydraulic fractures can be any number, modeling has been done from 1 fracture to 5 fractures. During the modeling of gas flow in the fracture, the gas flows from the matrix into the fracture.

4.1.2 Discretization Method:

The developed equation representing the gas flow in the hydraulic fracture is a nonlinear partial differential equation (PDE). For discretization of this nonlinear PDE, Finite difference method has been used.

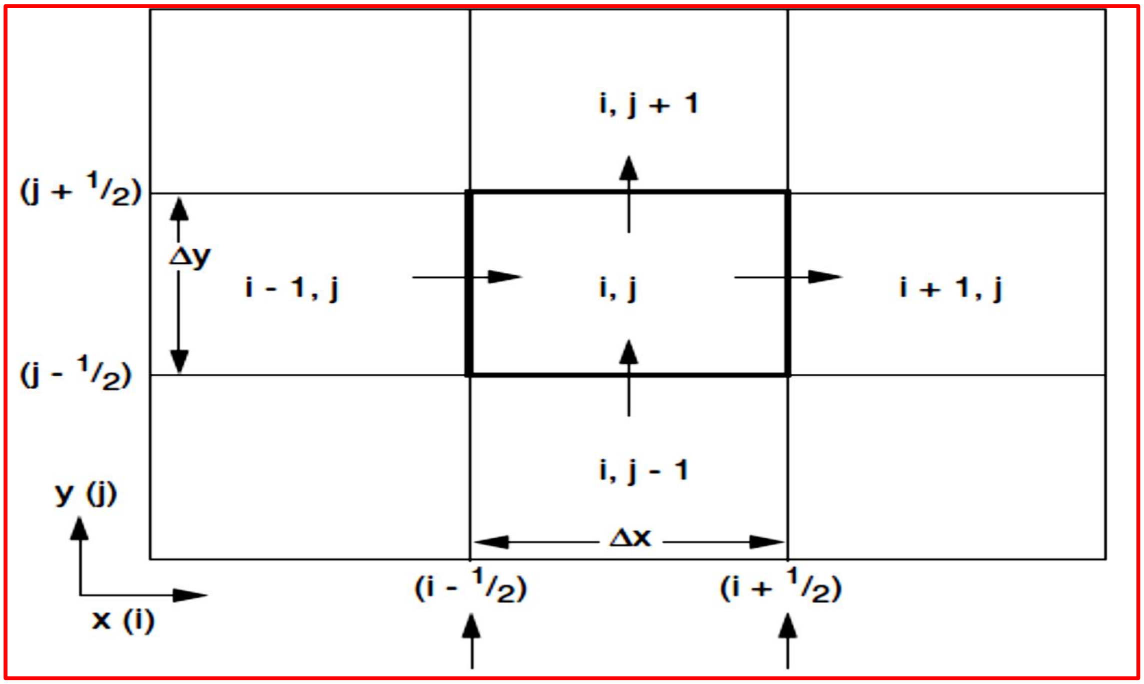


Figure 4.2: Discretization and notation indication for a 2D pressure equation for Matrix.

By applying finite difference method to equation 7, we get

$$\Rightarrow \left[\left(\frac{\beta_c\, k_{rgf}\; k}{\mu_{gf}} A_x\; \frac{\rho_{gsc}}{\alpha_c B_g}\frac{1}{\Delta x}\right)_{i+\frac{1}{2},j,k}\left(P^{n+1}_{i+1,j,k} - P^{n+1}_{i,j,k}\right) - \left(\frac{\beta_c\, k_{rgf}\; k}{\mu_{gf}} A_x\; \frac{\rho_{gsc}}{\alpha_c B_g}\frac{1}{\Delta x}\right)_{i-\frac{1}{2},j,k}\left(P^{n+1}_{i,j,k} - P^{n+1}_{i-1,j,k}\right)\right] + \left[\left[\left(\frac{\beta_c\, k_{rgf}\; k}{\mu_{gf}} A_y\; \frac{\rho_{gsc}}{\alpha_c B_g}\frac{1}{\Delta y}\right)_{i,j+\frac{1}{2},k}\left(P^{n+1}_{i,j+1,k} - P^{n+1}_{i,j,k}\right) - \left(\frac{\beta_c\, k_{rgf}\; k}{\mu_{gf}} A_y\; \frac{\rho_{gsc}}{\alpha_c B_g}\frac{1}{\Delta y}\right)_{i,j-\frac{1}{2},k}\left(P^{n+1}_{i,j,k} - P^{n+1}_{i,j-1,k}\right)\right]\right] + \left[\frac{-2}{\overline{w_f}}\frac{\beta_c k_{gm\Delta x\Delta y}}{\mu_{gm}}\frac{\rho_{gsc}}{\alpha_c B_g}\left(P^{n+1}_{i,j,k} - P^{n}_{i,j,k}\right)\right] = \left[\left(\frac{V_b \rho_{gsc}}{\alpha_c B_g}\right)\left(1 - S_{wf}\right)\emptyset_f c_f\right]\frac{\left(P^{n+1}_{i,j,k} - P^{n}_{i,j,k}\right)}{\Delta t} \text{ -----}\rightarrow \mathbf{8.}$$

As we have chosen the spatial discretization terms at new time level i.e. (n+1), the applied finite difference method can be considered has implicit finite difference method.

Considering transmissibility $(T_{gx}) = \frac{\beta_c\, k_{rg}}{\mu_g}\;\frac{\rho_{gsc}}{\alpha_{cB_g}}$

The eqn 8 can be written as

$$\Rightarrow \left[\left(\frac{K_f A_x T_{gx}}{\Delta x}\right)_{i+\frac{1}{2},j,k}\left(P_{i+1,j,k}^{n+1} - P_{i,j,k}^{n+1}\right) - \left(\frac{K_f A_x T_{gx}}{\Delta x}\right)_{i-\frac{1}{2},j,k}\left(P_{i,j,k}^{n+1} - P_{i-1,j,k}^{n+1}\right)\right] + \left[\left(\frac{K_f A_y T_{gy}}{\Delta y}\right)_{i,j+\frac{1}{2},k}\left(P_{i,j+1,k}^{n+1} - P_{i,j,k}^{n+1}\right) - \left(\frac{K_f A_y T_{gy}}{\Delta y}\right)_{i,j-\frac{1}{2},k}\left(P_{i,j,k}^{n+1} - P_{i,j-1,k}^{n+1}\right)\right] = \left[\left(\frac{2}{w_f} T_{gz}\Delta x \Delta y\right) + \left(\frac{V_b \rho_{gsc}}{\alpha_c B_g}\frac{(1-S_{wf})\emptyset_f c_f}{\Delta t}\right)\right]\left(P_{i,j,k}^{n+1} - P_{i,j,k}^{n}\right) \text{------}\rightarrow \textbf{9.}$$

$Writing\ eqn\ 9\ in\ the\ following\ form$

$$\lambda_{g_{i+\frac{1}{2},j,k}}\left(P_{i+1,j,k}^{n+1} - P_{i,j,k}^{n+1}\right) - \lambda_{g_{i-\frac{1}{2},j,k}}\left(P_{i,j,k}^{n+1} - P_{i-1,j,k}^{n+1}\right) + \lambda_{g_{i,j+\frac{1}{2},k}}\left(P_{i,j+1,k}^{n+1} - P_{i,j,k}^{n+1}\right) - \lambda_{g_{i,j-\frac{1}{2},k}}\left(P_{i,j,k}^{n+1} - P_{i,j-1,k}^{n+1}\right) = X_{i,j,k}\left(P_{i,j,k}^{n+1} - P_{i,j,k}^{n}\right) \text{------}\rightarrow \textbf{10.}$$

$Where,$

$$\lambda_{g_{i+\frac{1}{2},j,k}} = \left(\frac{K_f A_x T_{gx}}{\Delta x}\right)_{i+\frac{1}{2},j,k}$$

$$\lambda_{g_{i-\frac{1}{2},j,k}} = \left(\frac{K_f A_x T_{gx}}{\Delta x}\right)_{i-\frac{1}{2},j,k}$$

$$\lambda_{g_{i,j+\frac{1}{2},k}} = \left(\frac{K_f A_y T_{gy}}{\Delta y}\right)_{i,j+\frac{1}{2},k}$$

$$\lambda_{g_{i,j-\frac{1}{2},k}} = \left(\frac{K_f A_y T_{gy}}{\Delta y}\right)_{i,j-\frac{1}{2},k}.$$

$$X_{i,j,k} = \left[\left(\frac{2}{w_f} T_{gz}\Delta x \Delta y\right) + \left(\frac{V_b \rho_{gsc}}{\alpha_c B_g}\frac{(1-S_{wf})\emptyset_f c_f}{\Delta t}\right)\right]$$

$Eqn\ 10\ can\ be\ written\ in\ the\ form\ of$

$$B_{i,j,k}P_{i,j,k-1}^{n+1} + S_{i,j,k}P_{i,j-1,k}^{n+1} + W_{i,j,k}P_{i-1,j,k}^{n+1} + C_{i,j,k}P_{i,j,k}^{n+1} + E_{i,j,k}P_{i+1,j,k}^{n+1} + N_{i,j,k}P_{i,j+1,k}^{n+1} + A_{i,j,k}P_{i,j,k+1}^{n+1} = Q_{i,j,k}$$

Where,

$B_{i,j,k} = \lambda_{g_{i,j,k-\frac{1}{2}}}.$

$S_{i,j,k} = \lambda_{g_{i,j-\frac{1}{2},k}}.$

$W_{i,j,k} = \lambda_{g_{i-\frac{1}{2},j,k}}$

$$C_{i,j,k} = -\left[\lambda_{g_{i+\frac{1}{2},j,k}} + \lambda_{g_{i-\frac{1}{2},j,k}} + \lambda_{g_{i,j+\frac{1}{2},k}} + \lambda_{g_{i,j-\frac{1}{2},k}} + \lambda_{g_{i,j,k+\frac{1}{2}}} + \lambda_{g_{i,j,k-\frac{1}{2}}}. + X_{i,j,k}\right]$$

$E_{i,j,k} = \lambda_{g_{i+\frac{1}{2},j,k}}.$

$N_{i,j,k} = \lambda_{g_{i,j+\frac{1}{2},k}}$

$A_{i,j,k} = \lambda_{g_{i,j,k+\frac{1}{2}}}$

$Q_{i,j,k} = (-X_{i,j,k})P_{i,j,k}^{n}.$

4.2 <u>Material Balance Equation for flow of water in the Hydraulic Fracture:</u>

4.2.1 <u>Assumptions:</u>

1) The flow of water occurs only in X and Y directions only.
2) Width of the fracture is $\overline{w_f}$.
3) The entire flow process is isothermal. (T=Constant).
4) Hydraulic fracture is perpendicular to the horizontal wellbore.
5) The geometry of the hydraulic fracture is rectangle.

Material Balance Equation for the gas flow through the hydraulic fracture:

$(Accumulation\ with\ in\ system)$=$(Gas\ flow\ in\ through\ system\ boundaries)$-$(Gas\ flow\ out\ through\ system\ boundaries)$+$(Generation\ with\ in\ system)$-$(Consumption\ within\ system)$.

As control volume decreases, the volume = $\Delta x.\Delta y.\overline{w_f}$.

$$-\left(\upsilon_{w_{fx}}\rho_{w_f|_{x+\Delta x}}\Delta y.\overline{w_f}\right)-\left(-\upsilon_{w_{fx}}\rho_{w_f|_x}\Delta y.\overline{w_f}\right)+\left(-\upsilon_{w_{fy}}\rho_{w_f|_{y+\Delta y}}\Delta x.\overline{w_f}\right)-\left(-\upsilon_{w_{fy}}\rho_{w_f|_y}\Delta x.\overline{w_f}\right)=$$

$$\frac{\Delta\left((\Delta x\Delta y\overline{w_f}.\phi_f).\rho_{wf}.S_{wf}\right)}{\Delta t} \text{ ---------→ } \mathbf{11.}$$

Dividing equation 11 by Δx.Δy.$\overline{w_f}$, We got

$$\Rightarrow \frac{\left(-\upsilon_{w_{fx}}\,\rho_{wf}|_{x+\Delta x}+\upsilon_{w_{fx}}\,\rho_{wf}|_x\right)}{\Delta x}+\frac{\left(-\upsilon_{w_{fy}}\,\rho_{wf}|_{y+\Delta y}+\upsilon_{w_{fy}}\,\rho_{wf}|_y\right)}{\Delta y}=\frac{\Delta\left((\phi_f).\rho_{wf}.S_{wf}\right)}{\Delta t} \text{ ---------→ } \mathbf{12.}$$

Taking limit Δx , Δy and Δt ---→ 0,

$$\Rightarrow -\frac{\partial\left(\upsilon_{w_{fx}}\rho_{wf}\right)}{\partial x}-\frac{\partial\left(\upsilon_{w_{fy}}\rho_{wf}\right)}{\partial y}=\frac{\partial\left((\phi_f).\rho_{wf}.S_{wf}\right)}{\partial t} \text{ ---------→ } \mathbf{13.}$$

Considering Darcy's law, we have

$$\Rightarrow \upsilon_{w_{fx}}=\frac{-k_{wf}}{\mu_{wf}}\frac{\partial(P_{wf})}{\partial x} \text{ ---------→ for x-direction}$$

But, as the flow in the hydraulic fracture is two phase flow, the concept of relative permeability comes into act.

Relative permeability of gas with respect to water (k_{rwf})

$\Rightarrow$ $k_{rwf}=\frac{k_{wf}}{k_f}$ and

$\frac{k_{wf}}{k_f}\le 1.$

Now, $k_{wf}=k_{rwf}\ k_f$

Where, k_f= Absolute Permeability in fracture with proppant.[m^2].

k_{wf} = Effective permeability of gas in the fracture.

k_{rwf} = Relative permeability of gas in the fracture.

Now,

⇨ $-\upsilon_{w_{fx}} = \frac{-k_{rwf}\,k_f}{\mu_{wf}}\frac{\partial(P_{wf})}{\partial x}$ ----------------→ for x-direction.

⇨ $-\upsilon_{w_{fy}} = \frac{-k_{rwf}\;k_f}{\mu_{wf}}\frac{\partial(P_{wf})}{\partial y}$ ----------------→ for y-direction.

Substituting $\upsilon_{w_{fx}}$, $\upsilon_{w_{fy}}$ in equation 13, we get

⇨ $-\frac{\partial\left(\frac{-\beta_c\,k_{rwf}\;k}{\mu_{wf}}\frac{\partial(P_{wf})}{\partial x}\rho_{wf}\right)}{\partial x} - \frac{\partial\left(\frac{-\beta_c\,k_{rwf}\;k}{\mu_{wf}}\frac{\partial(P_{wf})}{\partial y}\rho_{wf}\right)}{\partial y} = \frac{\partial\left((\emptyset_f).\rho_{gf}.S_{gf}\right)}{\partial t}$-- -------------------→ **14.**

Where,

β_c= Transmissibility conversion factor = 1.127 $\frac{scf}{D-psi}$.

From equation of state,

Formation Volume Factor (B_w) = $\frac{\rho_{wsc}}{\alpha_c\rho_{wf}}$

From equation 14,

⇨ $\frac{\partial\left(\frac{\beta_c\,k_{rwf}\;k}{\mu_{wf}}\frac{\partial(P_{wf})}{\partial x}\frac{\rho_{wsc}}{\alpha_c B_w}\right)}{\partial x} + \frac{\partial\left(\frac{\beta_c\,k_{rwf}\;k}{\mu_{wf}}\frac{\partial(P_{wf})}{\partial y}\frac{\rho_{wsc}}{\alpha_c B_w}\right)}{\partial y} = \frac{\partial\left((\emptyset_f).\frac{\rho_{wsc}}{\alpha_c B_w}.S_{wf}\right)}{\partial t}$ ----------------------→ **15.**

From literature, $\emptyset_f = \emptyset_o e^{c_f\,(P_f - P_o\,)}$.

$\frac{\partial\emptyset_f}{\partial p_f} = \emptyset_f c_f$.

$$Now, \frac{\partial\emptyset_f}{\partial t} = \frac{\partial\emptyset_f}{\partial p_f}\frac{\partial p_f}{\partial t}$$

⇨ $\frac{\partial\emptyset_f}{\partial t} = \emptyset_f c_f\,\frac{\partial p_f}{\partial t}$.

Equation 15 can be rewritten as

⇨ $\frac{\partial\left(\frac{\beta_c\,k_{rwf}\;k}{\mu_{wf}}\frac{\partial(P_{wf})}{\partial x}\frac{\rho_{wsc}}{\alpha_c B_w}\right)}{\partial x} + \frac{\partial\left(\frac{\beta_c\,k_{rwf}\;k}{\mu_{wf}}\frac{\partial(P_{wf})}{\partial y}\frac{\rho_{wsc}}{\alpha_c B_w}\right)}{\partial y} = \left[\left(\frac{\rho_{wsc}}{\alpha_c B_{gw}}\right)S_{wf}\emptyset_f c_f\right]\frac{\partial p_f}{\partial t}$---------------→ **16.**

Dividing equation 6 with$\Delta x.\Delta y.\Delta z$, we get

$$\Rightarrow \frac{\partial\left(\frac{\beta_c k_{rwf} k}{\mu_{wf}} A_x \frac{\partial(P_{wf})}{\partial x} \frac{\rho_{wsc}}{\alpha_c B_w}\right)}{\partial x} \Delta x + \frac{\partial\left(\frac{\beta_c k_{rwf} k}{\mu_{wf}} A_y \frac{\partial(P_{wf})}{\partial y} \frac{\rho_{wsc}}{\alpha_c B_w}\right)}{\partial y} \Delta y = \left[\left(\frac{V_b \rho_{wsc}}{\alpha_c B_w}\right)(1 - S_{gf}) \emptyset_f c_f\right] \frac{\partial p_f}{\partial t} \quad \text{--}\rightarrow \mathbf{17.}$$

In this case, the flow is a multiphase flow i.e. gas and water will flow through the fracture. So, the saturation values of gas and water will change with respect to time during production. Initially, water saturation will be nearly 100% and then the water saturation decreases and the gas saturation increases. A well bore has been considered in the 5th layer from the top of the reservoir. As the hydraulic fractures can be any number, modeling has been done from 1 fracture to 5 fractures. Equation 17 represents the flow of water in the fracture during gas production from shale reservoirs.

4.2.2 Discretization Method:

The developed equation representing the gas flow in the hydraulic fracture is a nonlinear partial differential equation (PDE). For discretization of this nonlinear PDE, Finite difference method has been used.

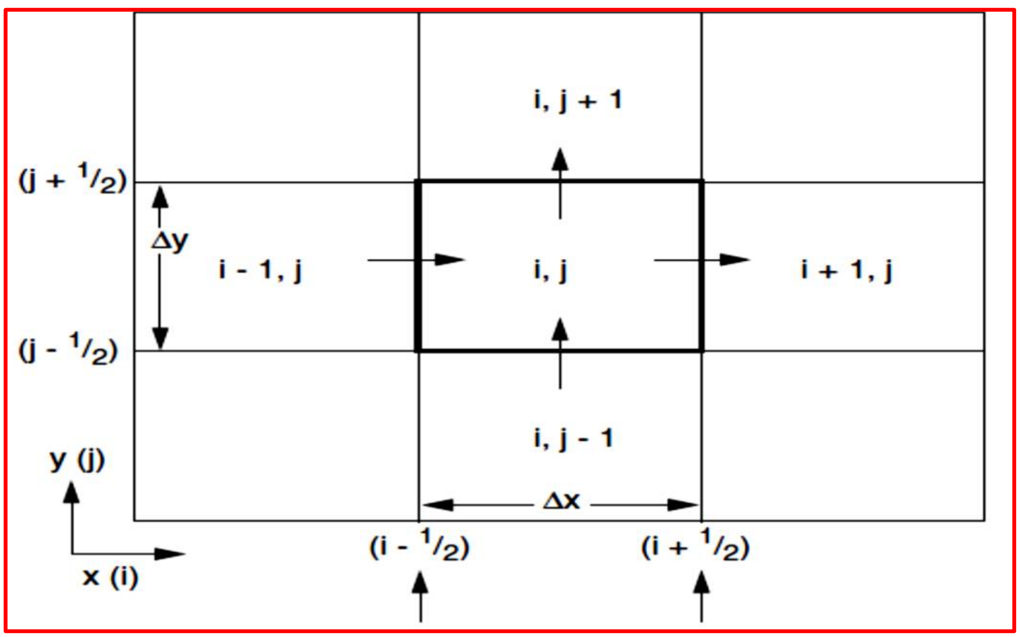


Figure 4.3: Discretization and notation indication for a 2D pressure equation for Hydraulic Fracture.

As we have chosen the spatial discretization terms at new time level i.e. (n+1), the finite difference method applied here is implicit finite difference method.

By applying finite difference method to equation 17, we get

$$\Rightarrow \left[\left(\frac{\beta_c\, k_{rwf}\ k}{\mu_{wf}} A_x\ \frac{\rho_{wsc}}{\alpha_c B_w}\frac{1}{\Delta x}\right)_{i+\frac{1}{2},j,k}\left(P^{n+1}_{i+1,j,k} - P^{n+1}_{i,j,k}\right) - \left(\frac{\beta_c\, k_{rwf}\ k}{\mu_{wf}} A_x\ \frac{\rho_{wsc}}{\alpha_c B_w}\frac{1}{\Delta x}\right)_{i-\frac{1}{2},j,k}\left(P^{n+1}_{i,j,k} - P^{n+1}_{i-1,j,k}\right)\right] + \left[\left[\left(\frac{\beta_c\, k_{rwf}\ k}{\mu_{wf}} A_y\ \frac{\rho_{wsc}}{\alpha_c B_w}\frac{1}{\Delta y}\right)_{i,j+\frac{1}{2},k}\left(P^{n+1}_{i,j+1,k} - P^{n+1}_{i,j,k}\right) - \left(\frac{\beta_c\, k_{rwf}\ k}{\mu_{wf}} A_y\ \frac{\rho_{wsc}}{\alpha_c B_w}\frac{1}{\Delta y}\right)_{i,j-\frac{1}{2},k}\left(P^{n+1}_{i,j,k} - P^{n+1}_{i,j-1,k}\right)\right]\right] = \left[\left(\frac{V_b \rho_{wsc}}{\alpha_c B_w}\right)\left(1 - S_{gf}\right)\emptyset_f c_f\right]\frac{\left(P^{n+1}_{i,j,k} - P^{n}_{i,j,k}\right)}{\Delta t} \text{ - } \text{-----}\rightarrow \mathbf{18.}$$

Considering transmissibility $(T_{wx}) = \frac{\beta_c\, k_{rw}}{\mu_w}\ \frac{\rho_{wsc}}{\alpha_{cB_w}}$

The eqn 18 can be written as

$$\Rightarrow \left[\left(\frac{K_f A_{xT_{wx}}}{\Delta x}\right)_{i+\frac{1}{2},j,k}\left(P^{n+1}_{i+1,j,k} - P^{n+1}_{i,j,k}\right) - \left(\frac{K_f A_{xT_{wx}}}{\Delta x}\right)_{i-\frac{1}{2},j,k}\left(P^{n+1}_{i,j,k} - P^{n+1}_{i-1,j,k}\right)\right] + \left[\left(\frac{K_f A_{yT_{wy}}}{\Delta y}\right)_{i,j+\frac{1}{2},k}\left(P^{n+1}_{i,j+1,k} - P^{n+1}_{i,j,k}\right) - \left(\frac{K_f A_{yT_{wy}}}{\Delta y}\right)_{i,j-\frac{1}{2},k}\left(P^{n+1}_{i,j,k} - P^{n+1}_{i,j-1,k}\right)\right] = \left[\left(\frac{V_b \rho_{gsc}}{\alpha_c B_g}\frac{(1-S_{wf})\emptyset_f\, c_f}{\Delta t}\right)\right]\left(P^{n+1}_{i,j,k} - P^{n}_{i,j,k}\right) \text{-----}\rightarrow \mathbf{19.}$$

Writing eqn 19 in the following form

$$\lambda_{w_{i+\frac{1}{2},j,k}}\left(P^{n+1}_{i+1,j,k} - P^{n+1}_{i,j,k}\right) - \lambda_{w_{i-\frac{1}{2},j,k}}\left(P^{n+1}_{i,j,k} - P^{n+1}_{i-1,j,k}\right) + \lambda_{w_{i,j+\frac{1}{2},k}}\left(P^{n+1}_{i,j+1,k} - P^{n+1}_{i,j,k}\right) - \lambda_{w_{i,j-\frac{1}{2},k}}\left(P^{n+1}_{i,j,k} - P^{n+1}_{i,j-1,k}\right) = X_{i,j,k}\left(P^{n+1}_{i,j,k} - P^{n}_{i,j,k}\right) \text{-----}\rightarrow \mathbf{20.}$$

Where,

$$\lambda_{w_{i+\frac{1}{2},j,k}} = \left(\frac{K_f A_{xT_{wx}}}{\Delta x}\right)_{i+\frac{1}{2},j,k}$$

$$\lambda_{w_{i-\frac{1}{2},j,k}} = \left(\frac{K_f A_{xT_{wx}}}{\Delta x}\right)_{i-\frac{1}{2},j,k}$$

$$\lambda_{w_{i,j+\frac{1}{2},k}} = \left(\frac{K_f A_{yT_{wy}}}{\Delta y} \right)_{i,j+\frac{1}{2},k}$$

$$\lambda_{w_{i,j-\frac{1}{2},k}} = \left(\frac{K_f A_{yT_{wy}}}{\Delta y} \right)_{i,j-\frac{1}{2},k}.$$

$$X_{i,j,k} = \left[\left(\frac{V_b \rho_{wsc}}{\alpha_c B_w} \frac{(1-S_{gf}) \emptyset_f c_f}{\Delta t} \right) \right]$$

$Eqn\ 20\ can\ be\ written\ in\ the\ form\ of$

$$S_{i,j,k} P_{i,j-1,k}^{n+1} + W_{i,j,k} P_{i-1,j,k}^{n+1} + C_{i,j,k} P_{i,j,k}^{n+1} + E_{i,j,k} P_{i+1,j,k}^{n+1} + N_{i,j,k} P_{i,j+1,k}^{n+1} = Q_{i,j,k}$$

Where,

$$S_{i,j,k} = \lambda_{g_{i,j-\frac{1}{2},k}}.$$

$$W_{i,j,k} = \lambda_{g_{i-\frac{1}{2},j,k}}$$

$$C_{i,j,k} = -\left[\lambda_{g_{i+\frac{1}{2},j,k}} + \lambda_{g_{i-\frac{1}{2},j,k}} + \lambda_{g_{i,j+\frac{1}{2},k}} + \lambda_{g_{i,j-\frac{1}{2},k}} + X_{i,j,k} \right]$$

$$E_{i,j,k} = \lambda_{g_{i+\frac{1}{2},j,k}}.$$

$$N_{i,j,k} = \lambda_{g_{i,j+\frac{1}{2},k}}$$

$$Q_{i,j,k} = \left(-X_{i,j,k} \right) P_{i,j,k}^{n}.$$

As, in a shale gas reservoir there may be N number of hydraulic fractures in order to increase the interconnectivity of the wellbore with the matrix, which will increase the rate of production. In this model, we have done my simulation starting with single hydraulic fracture and continued till 5 hydraulic fractures. The pictorial representation of the different positions of hydraulic fractures is shown in Figure 4.4(a,b,c and d), generally equispaced.

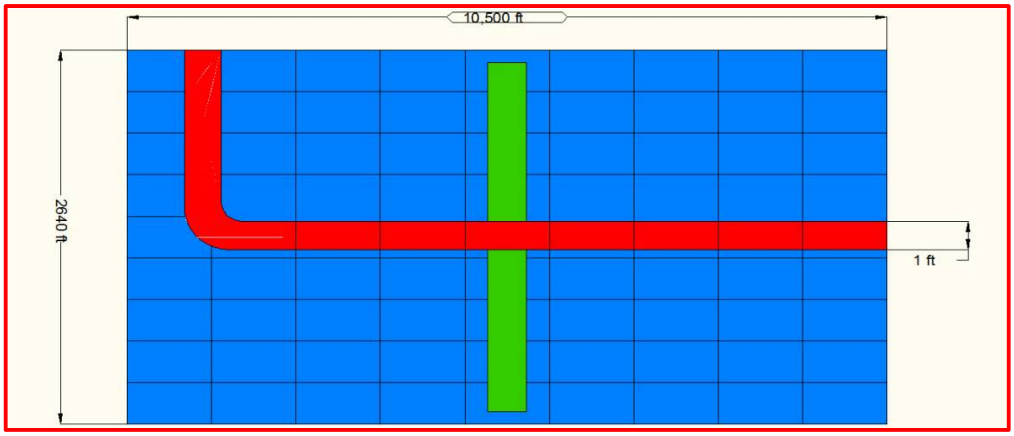


Figure 4.4(a): Schematic representation of a Single Hydraulically Fractured Reservoir.

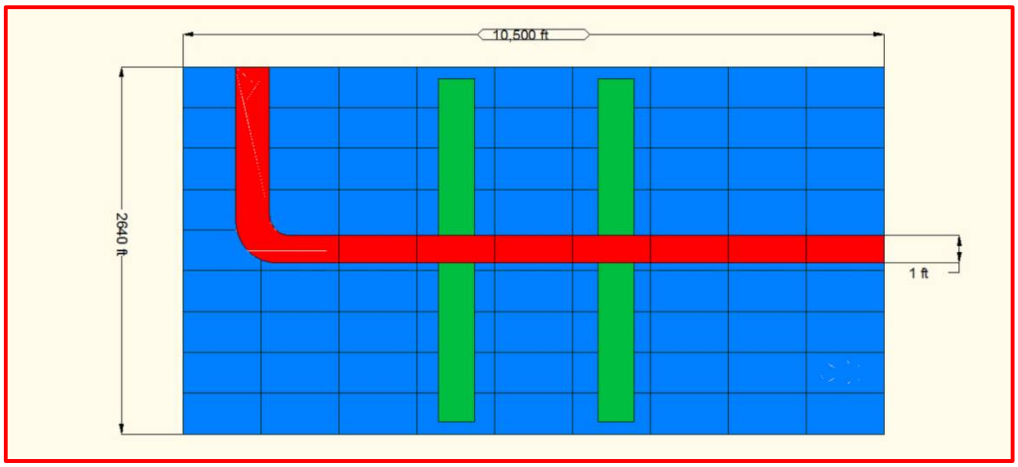


Figure 4.4(b): Schematic representation of a Two Hydraulically Fractured Reservoir.

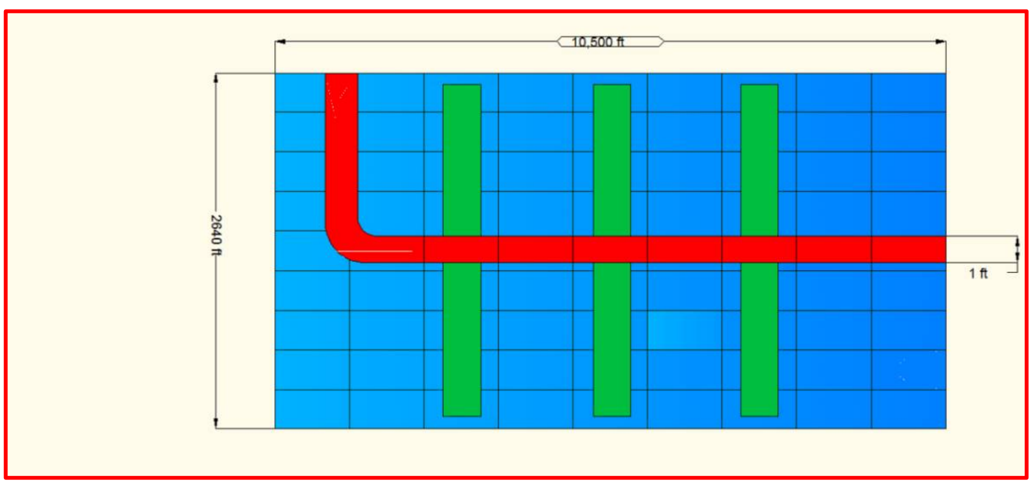


Figure 4.3(c): Schematic Representation of a Three Hydraulically Fractured Reservoir.

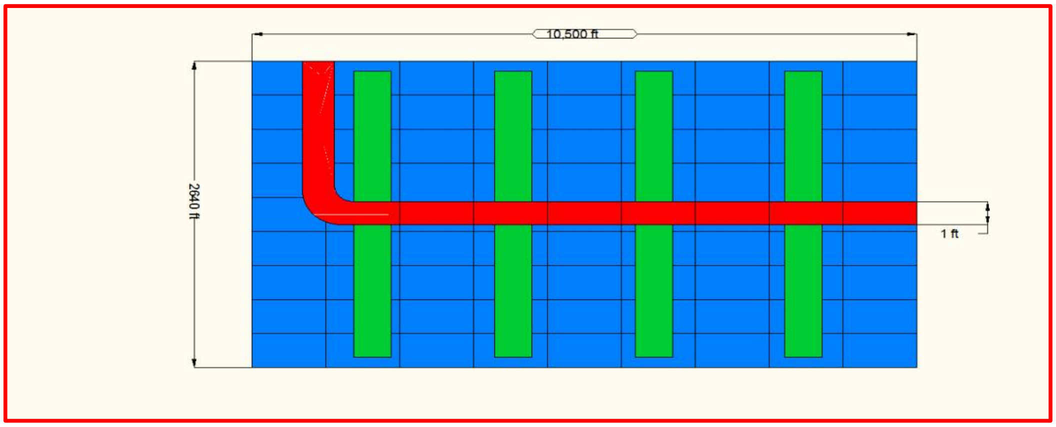
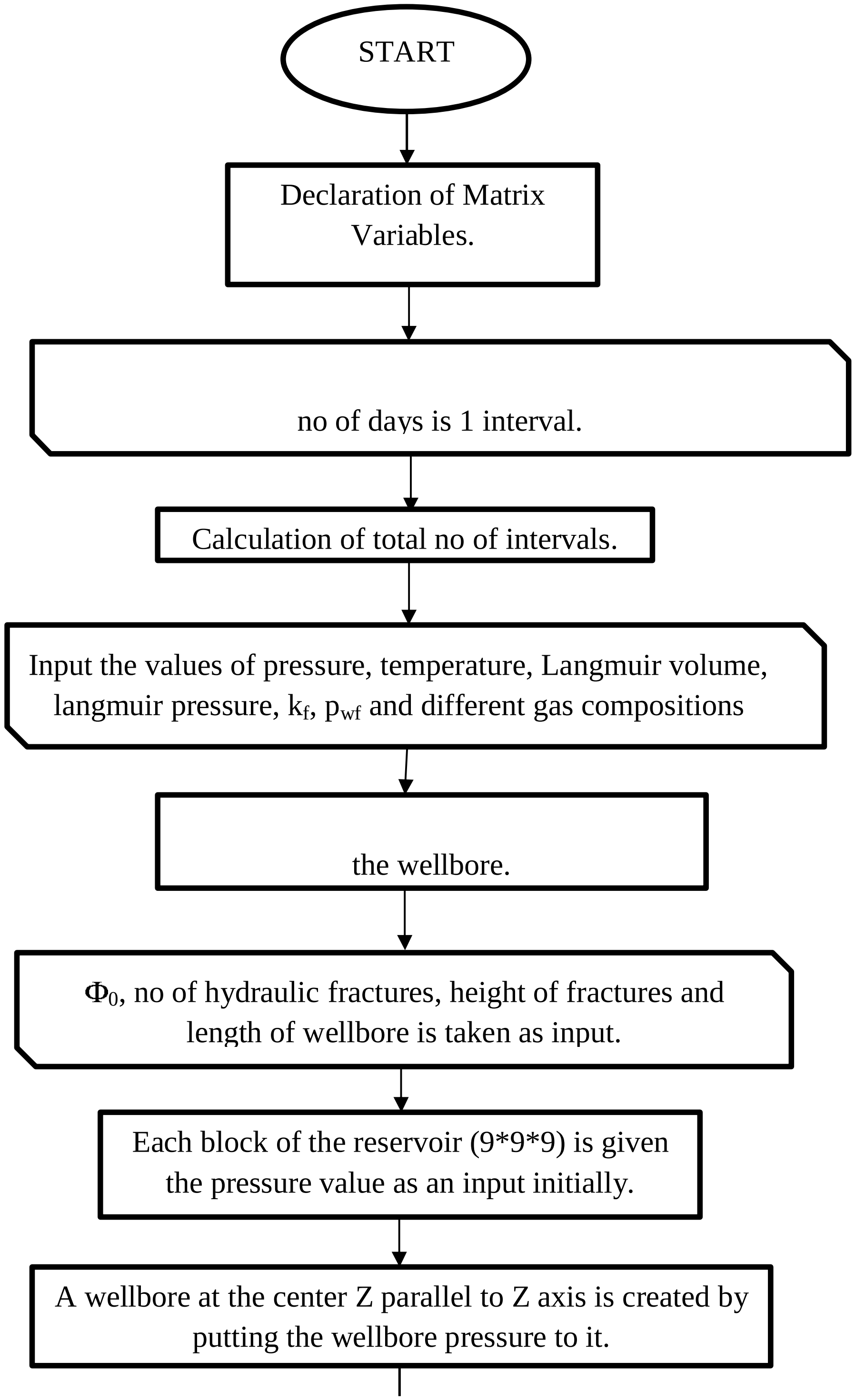


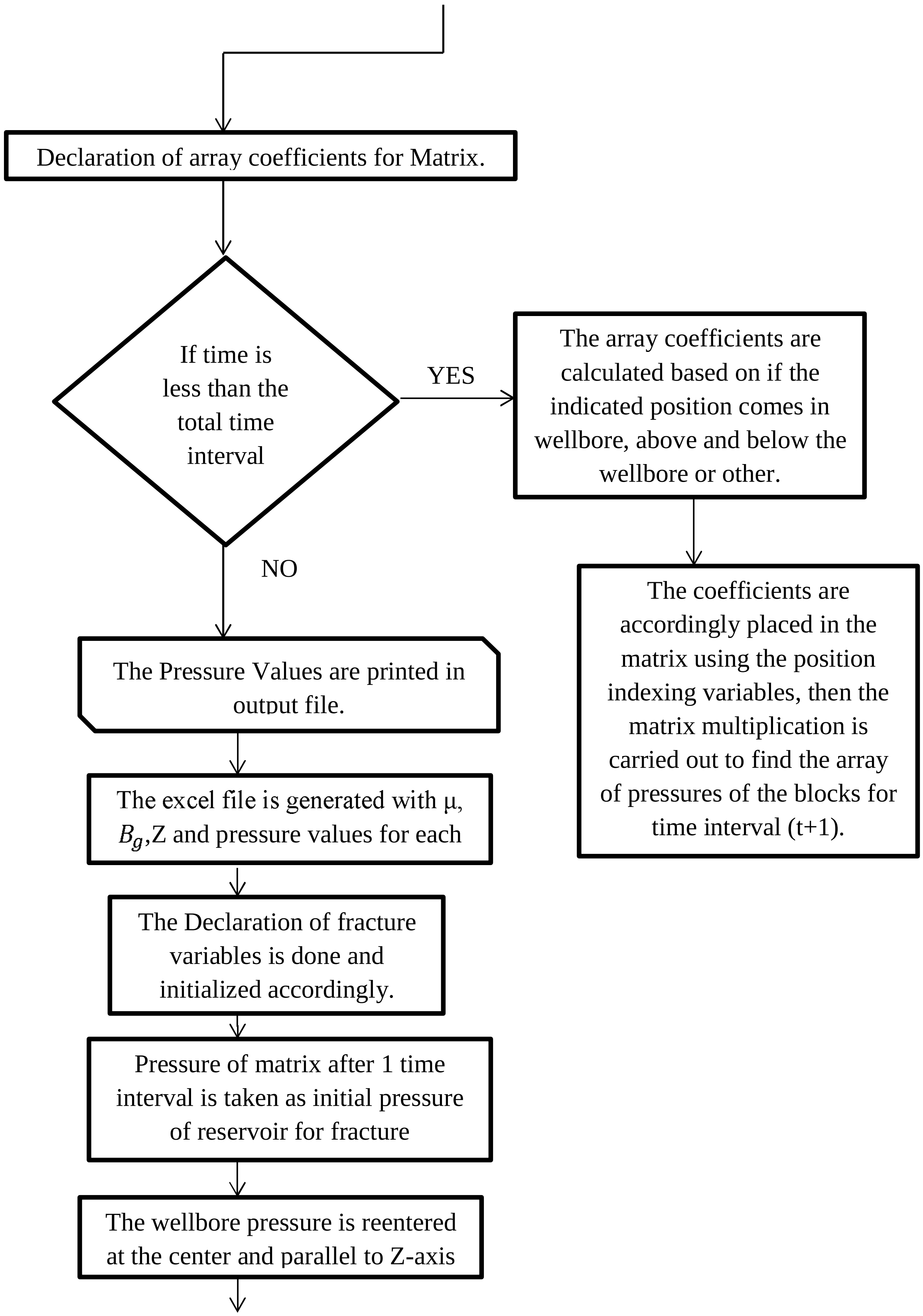
Figure 4.4(d): Schematic Representation of a Four Hydraulically Fractured Reservoir.

As the hydraulic fractures are surrounded by matrix blocks and the flow of gas will be initially from the matrix to the fracture, while simulating the hydraulic fractured reservoir the non-linear PDE's of both the matrix and the fractures are simultaneously solved for getting the flow rates of gas from the hydraulic fracture to the wellbore.

4.3 **Algorithm:**

The developed nonlinear partial differential equation is compiled using JAVA for determining the pressure variation in the hydraulically fractured blocks during gas production from shale reservoirs. Figure 4.5 represents the algorithm, used for solving the nonlinear PDE for gas flow inside the matrix.


START
Declaration of Matrix Variables.
no of days is 1 interval.
Calculation of total no of intervals.
Input the values of pressure, temperature, Langmuir volume, langmuir pressure, k_f, p_{wf} and different gas compositions
the wellbore.
Φ_0, no of hydraulic fractures, height of fractures and length of wellbore is taken as input.
Each block of the reservoir (9*9*9) is given the pressure value as an input initially.
A wellbore at the center Z parallel to Z axis is created by putting the wellbore pressure to it.


Declaration of array coefficients for Matrix.
If time is less than the total time interval
YES
The array coefficients are calculated based on if the indicated position comes in wellbore, above and below the wellbore or other.
The coefficients are accordingly placed in the matrix using the position indexing variables, then the matrix multiplication is carried out to find the array of pressures of the blocks for time interval (t+1).
NO
The Pressure Values are printed in output file.
The excel file is generated with μ, B_g,Z and pressure values for each
The Declaration of fracture variables is done and initialized accordingly.
Pressure of matrix after 1 time interval is taken as initial pressure of reservoir for fracture
The wellbore pressure is reentered at the center and parallel to Z-axis

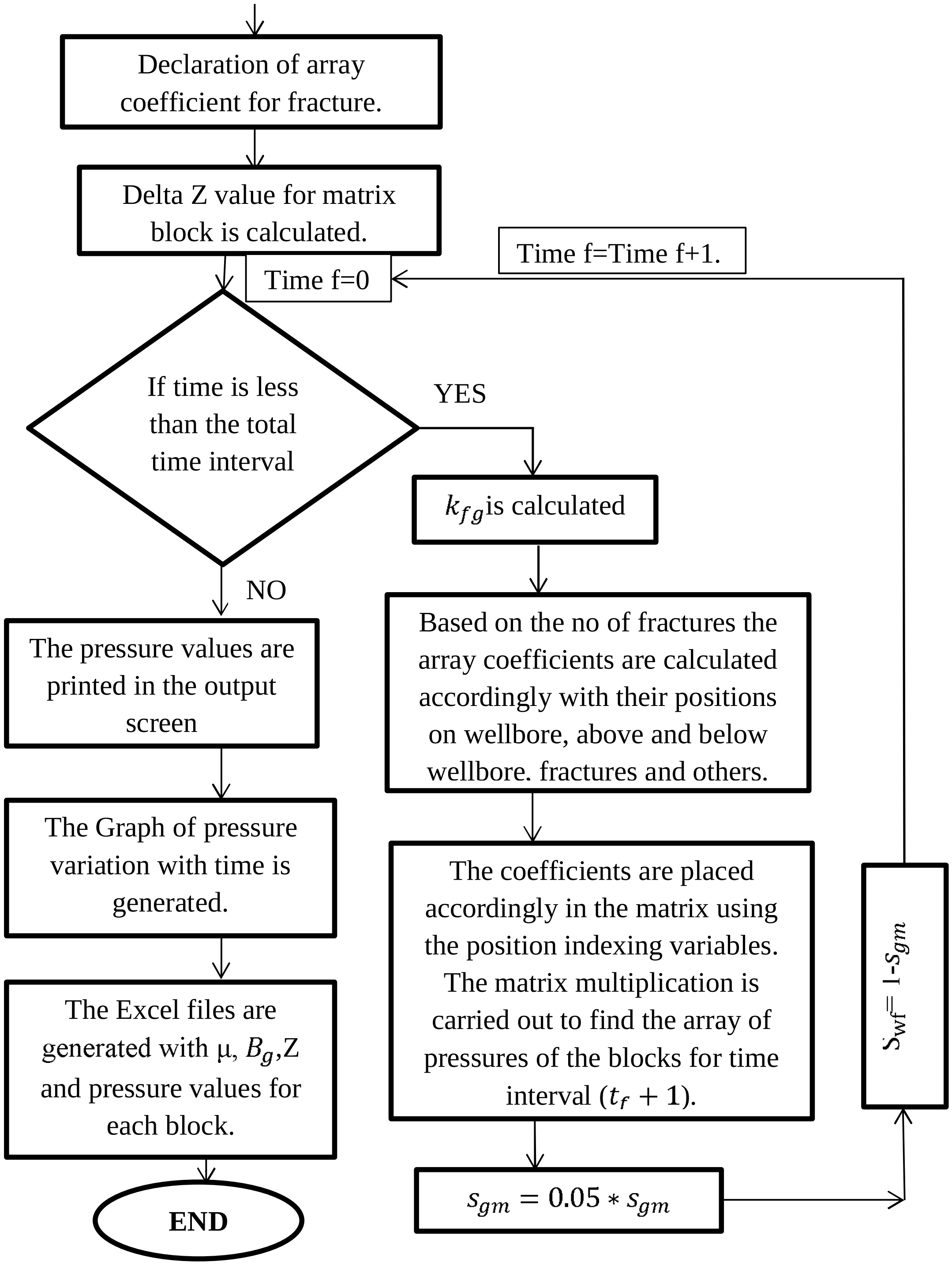


Figure 4.5: Algorithm representing the procedure for solving the gas flow in the hydraulic fractures.

The derived nonlinear partial differential equation for flow of gas in induced or hydraulic fractures is compiled using JAVA programming language and the code is attached in ANNEXURE-II.