CHAPTER 2

CHAPTER 2

RECEIVER FUNCTION METHOD

2.1 INTRODUCTION

Receiver function is an effective technique for extracting information of crust and upper mantle structure. It is primarily used to estimate the thickness of different subsurface layers based on the seismic shear wave velocities estimated at different depths (Burdick & Langston, 1977; Vinnik, 1977; Langston, 1979). The technique makes use of teleseismic body waveforms for computation of RF. The RFs are time series computed from three-component seismograms that contain relative response of the earth near the receiver site. The incoming teleseismic wave bears the information of the earthquake source, the path traveled by the seismic wave and response of local earth's structure beneath the receiver site. The RF is obtained by removing the effects of source, the travel path and instrument response by deconvolving vertical component seismogram with the radial and tangential components resulting radial receiver function (RRF) and tangential receiver function (TRF), respectively. The foundation of RF method is based on that the incoming near vertical P- wave gets converted into S- wave when encounters a velocity discontinuity like Moho discontinuity. A part of this P-to-S (Ps) converted phase gets reflected that causes reverberated phases or crustal multiples (e.g. $PpPs$, $PsSs+ PsPs$, $PpPs$) (Fig. 2.1a, b). Since S- wave travels at the slower velocity than P- wave, the converted Ps phases arrive within the P- wave coda following the direct P onset (Pp phase). The difference in velocities between Ps and Pp phases provides a clue to estimate the thickness of the converting layer.

Since a long time, teleseismic body waves have been used to retrieve crustal and lithospheric structures beneath recording stations under the name of crustal transfer method (Phinney, 1964). The method has subsequently been improved by several workers (e.g. Clayton & Wiggins, 1976; Burdick Langston, 1977; Vinnik, 1977; Vinnik & Kosarev, 1981) and finally known as RF method. Clayton and Wiggins, (1976) have introduced spectral water level deconvolution technique. Owens et al. (1984) have also improved the RF method developing linearized time-domain inversion routine for obtaining shear velocity model.
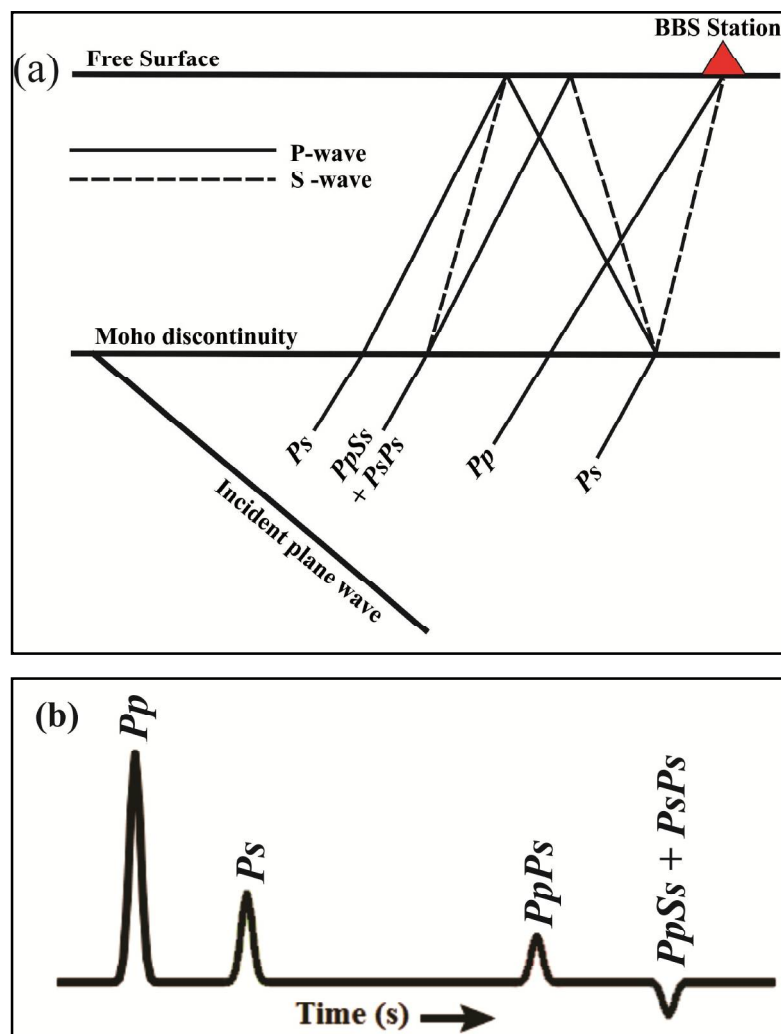

Figure 2.1 (a) Simplified ray diagram of the P-to-S (*Ps*) converted phase and its crustal multiples *(PpPs, PpSs + PsPs)* for a simple earth model with one layer and (b) the corresponding theoretical receiver function.

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Prior to 1991, most of the studies used normalized RFs. Ammon, (1991) introduced a modified deconvolution technique that preserves the absolute amplitude of RFs for constraining near-surface velocity structure. A modified deconvolution method was introduced by Sheehan et al. (1995) using a damping function in place of water level (Clayton & Wiggins, 1976). The deconvolution method was further improved by Ligorria and Ammon, (1999) using iterative time-domain deconvolution technique for RF estimation. Nowadays, this method is widely used as it gives better results for data with a highsignal-to-noise ratio (SNR). Besides, these methods, there are several other methods like multiple-taper correlation (MTC) (Vernon et al., 1991; Park & Levin, 2000) and extended-time multitaper frequency domain approach (Helffrich, 2006).

The RF method is not only used for estimating crustal thickness variations (e.g. Zhu & Kanamori, 2000; Gilbert & Sheehan, 2004; Lodge & Helffrich, 2006; Rai et al., 2006; Oreshin et al., 2008, 2011; Hazarika et al., 2013, 2014) but also used for mapping the nature of intra-crustal features (e.g. Schulte-Pelkum et al., 2005; Gashawbeza et al., 2008; Caldwell et al., 2013), imaging subducting lithosphere, both active and fossil (e.g. Bostock, 1998; Bostock & Rondenay, 1999; Kosarev et al., 1999; Park et al., 2004; Nabelek et al., 2009), transition zone interfaces (Gurrola & Minster, 1998; Chevrot et al., 1999; Nyblade et al., 2000), dipping structure (Langston, 1979; Owens et al., 1984; Cassidy, 1992; Zhang & Langston, 1995) and anisotropic layers (Levin & Park, 1997, 2000).

In this chapter, P-wave RF analysis method has been discussed which was adopted for crustal structure study for the study area.

2.2 MATHEMATICAL APPROACH

The seismic waveforms contain information related to the instrument response, earthquake source time function and response of the propagating media. Mathematically, displacement or velocity response $Y(t)$ of a teleseismic waveform arriving at a distant station can be represented as the convolution of three basic filters by the following equation:

$$Y(t) = I(t)*S(t)*E(t) \qquad (2.1)$$

where $\quad I(t)$ is the impulse response of the recording instrument;

$S(t)$ is effective seismic source function;

$E(t)$ is the impulse response of the local earth structure and operator (*) represents convolution.

The digital 3-component seismograph records ground vibrations in three orthogonal components viz. vertical (Z), radial (R) and transverse (T). The orthogonal components of the ground motion can be written as:-

$$Y_V(t) = I(t)*S(t)*E_V(t)$$
$$Y_R(t) = I(t)*S(t)*E_R(t) \qquad (2.2)$$
$$Y_T(t) = I(t)*S(t)*E_T(t)$$

Where subscripts V, R and T represent vertical, radial and tangential components, respectively.

The time domain expressions of eqn. (2.2) can be written in the form of frequency domain as:-

$$Y_V(\omega) = I(\omega)S(\omega)E_V(\omega)$$
$$Y_R(\omega) = I(\omega)S(\omega)E_R(\omega) \qquad (2.3)$$
$$Y_T(\omega) = I(\omega)S(\omega)E_T(\omega)$$

Where $Y_V(\omega)$, $Y_R(\omega)$, and $Y_T(\omega)$ are the Fourier transform of $Y_V(t)$, $Y_R(t)$ and $Y_T(t)$, respectively. The convolution is simple multiplication in frequency domain.

In RF method, the P-wave and its multiples dominate the vertical component, while the Ps conversion and its multiples are registered prominently on the horizontal radial (SV) component. In order to observe the Ps conversion clearly in radial component, the instrument response and the source effect are eliminated by simple division in frequency domain to produce radial RF (RRF) which is mathematically expressed as:-

$$RRF(\omega) = \frac{Y_R(\omega)}{Y_V(\omega)} = \frac{E_R(\omega)}{E_V(\omega)} \qquad (2.4)$$

The vertical component of ground motion for a steeply incident P wave (in the epicentral distance used for RF analysis) consists of large arrivals followed by only small arrivals due to crustal reverberations and phase conversion. Thus vertical component is assumed to contain source and path effects as well as not significantly contaminated by near receiver structure. Therefore, $E_V(t)$ can be approximated as a Dirac delta function.

$$E_V(t) \sim \delta(t)$$

thus

$$RRF(\omega) \sim E_R(\omega) \qquad (2.5)$$

This eqn. 2.5 suggests that the RRF is approximately equal to the radial impulse response of the earth structure and similarly tangential RF (TRF) is the tangential component of the impulse response of the earth which can be expressed as:

$$TRF(\omega) \sim E_T(\omega) \qquad (2.6)$$

The RF computation in frequency domain involves simple division. While stable for noise free data, this divisional deconvolution is numerically unstable due to the spectral division with very small numbers and the band limited seismograms containing noise. These instabilities are removed by reducing high-frequency noise by convolving the resulting time series with a low pass Gaussian filter, $G(w)$. The Gaussian filter is selected as it is simple in shape with zero phase distortion and absence of side lobes.

Incorporating the Gaussian filter parameter $G(\omega)$, the deconvolution as shown in eqn.2.4 can be written as

$$RRF = \frac{Y_R(w)\overline{Y_V(w)}}{} \bullet G(\omega) \qquad (2.7)$$

where,

$$\varphi(\omega) = \max \left\{ Y_V(\omega)\overline{Y_V(\omega)}, c.\max [Y_V(\omega)\overline{Y_V(\omega)}] \right\} \qquad (2.8)$$

and

$$G(\omega) = \exp(-\omega^2/4*\alpha^2)$$

where, $\overline{Yv(\omega)}$ indicates the complex conjugate of Y_V, constant c in eqn. 2.8 represents minimum amplitude allowed in the denominator of eqn. 2.7 and a is the Gaussian width factor which governs the pass-band of the Gaussian filter, and therefore the width of the Gaussian pulse in the time domain. The Gaussian width parameter a, is chosen to exclude the high-frequency noise or scattered energy. The parameter "c" is a water level parameter that normalizes the Gaussian function to unit amplitude in the time domain. The value of c is selected as small as possible for a better resolution and noise level. Transforming RRF(ω) and TRF(ω) in the time domain produces the estimates of RRF(t) and TRF(t), respectively.

2.2.1 WATER LEVEL DECONVOLUTION

The water level deconvolution is the simplest deconvolution method used for computation of RF (Fig. 2.2a, b). It is a frequency domain deconvolution of the radial component with horizontal component shown in eqn. 2.7. This is done to avoid numerical instabilities in deconvolution process. In water-level deconvolution, replacing the small values in the denominator with a fraction of the maximum value of the denominator amplitude helps in avoiding the division
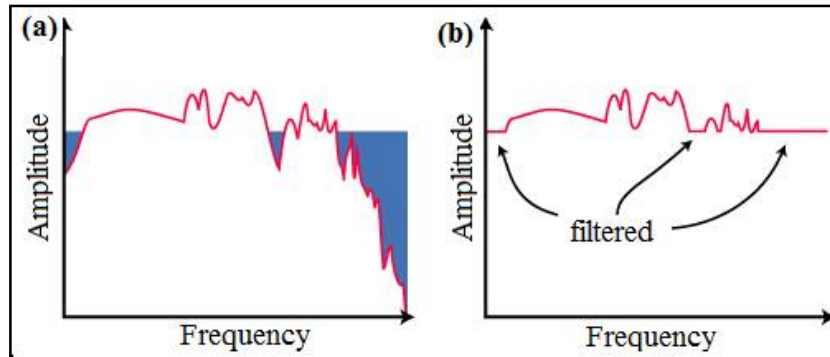

Figure 2.2 Illustration of water level deconvolution showing (a) amplitude spectrum with water level highlighted by blue colour, (b) amplitude spectrum after water level deconvolution.

by small numbers. This fraction value is termed as the water-level parameter, which is chosen by hit and trial method. In practice, a range of "*c*" values within 1.0 to 0.0001 is used during deconvolution. The appropriate value of "*c*" is controlled by the SNR and type of the vertical component seismogram is used and selected by scrutinizing the results of numerous trial runs of water-level fractions.

The acceptable noise levels in the RFs are produced by the lowest water level. Since water level filter can cause distortions of the RF, it is generally attempted to keep the values smaller in order to obtain better results.

2.2.2 ITERATIVE TIME DOMAIN DECONVOLUTION

Iterative deconvolution is a time domain deconvolution technique which was initially used for estimation of earthquakes source time function (Kikuchi & Kanamori, 1982) and later adopted for RF computation (Ammon et al., 1990). The detailed mathematical basis of this method is illustrated in Kikuchi and Kanamori, (1982). The assumption behind this is that the radial seismogram is a convolution of the vertical component with the earth's structure. When convolved with the vertical component, the earth's structure can be obtained by pursuing a time-series that would approximate its corresponding horizontal component.

RF is a time series that primarily defines the impulse response of the earth and can be interpreted mathematically as linear combinations of Dirac delta function $\delta(t)$ shifted in time as shown below

$$\text{RF}(t) = \sum_{i=1}^{n} x_i \delta(t - t_i) \qquad (2.9)$$

where, x_i is the amplitude of Dirac delta function and t_i defines the time with i^{th} iteration.

The iterative deconvolution approach is based on the least-squares minimization of the difference between a predicted signals generated by the convolution of an iteratively updated spike train with the vertical-component seismogram and the actual horizontal seismogram observed. This method uses cross-correlation function for estimation of the time, amplitude and lag of the

spikes that create the final RF. The steps involved in each iteration are as follows:

I.   Using cross-correlation of two different time series i.e. radial component (R^{i-1}) and a vertical component (V) of the recorded seismogram, time t_i is estimated having maximum amplitude x_i' in the cross-correlallogram.

II.   The scalar coefficient x_i is obtained with the best fit of time-shifted vertical component (V) with radial component (R^{i-1}) by minimizing the quantity "C" defined by $C = |R^i - x_i V * \delta (t - t_i)|^2$, where "*" is the convolution operator and $x_i = x_i'/V.V$ ("." represents zero time cross-correlation).

III.   Dirac delta function of amplitude x_i at time t_i is added to the current RF and residual R^i is calculated from R^{i-1} using the relation

$$R^i = R^{i-1} - x_i V * \delta (t - t_i) \qquad (2.10)$$

IV.   The above process is repeated till the minimum misfit is obtained. The misfit of the residual component, R_i, obtained from eqn. 2.10 is reduced and iteration comes to an end when the misfit between the obtained RF convolved with the vertical and radial component of the seismogram becomes negligible.

Iterative deconvolution procedure results in RFs having very less amount of deconvolution induced noise similar to those detected in the water-level deconvolution procedure (Fig. 2.3a, b). Moreover, the complex relationship among water-level values, time-domain smoothing, damping parameters, and the resulting RF can also be avoided by means of this method (Ligorria& Ammon, 1999). This deconvolution technique has been used in the present study.
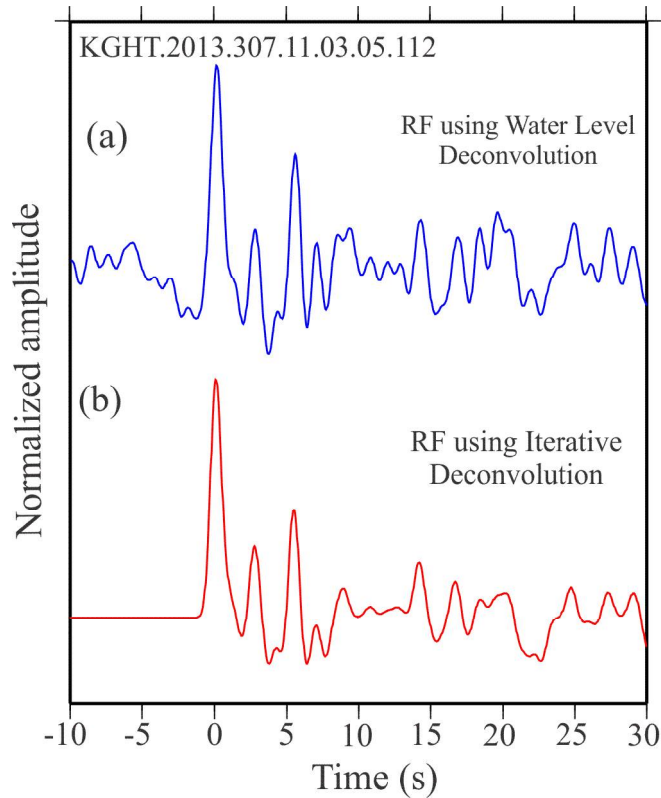
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Figure 2.3 Comparison of RFs computed using (a) Water-level deconvolution and (b) Iterative deconvolution technique.

2.3 RECEIVER FUNCTION COMPUTATION

The data processing steps applied for RF computation are discussed here.

2.3.1 PRE-PROCESSING OF DATA

The raw data from field stations, which are originally in MiniSeed format, is converted to SAC format, which is a binary data format of Seismic Analyzing Code (SAC) software (Goldstein et al., 2003). SAC is a flexible seismic data processing package developed at Lawrence Livermore National Laboratory, California, United States (Goldstein et al., 2003). The raw data obtained from field seismological stations are pre-processed in SAC software. Furthermore, in RF computation, the computer programs use waveforms with SAC data format. The necessary headers for the SAC format data files are filled up with the hypocentral information of earthquakes e.g. date, origin time, latitude, longitude, depth, and magnitude along with information of seismological

stations (e.g. latitude, longitude, and elevation). The hypocentral information of the teleseismic earthquakes is obtained from the catalog of United States Geological Survey (USGS; http://neic.usgs.gov). The quality of the digital data is checked visually and only those events are selected for further analysis for which SNR is high. It has been observed that some seismograms are not time synchronized. Therefore, prior to RF computation, the seismograms are time-synchronized. The mean and trend present in the digital waveform data are removed with the help of SAC software. The original waveforms are windowed in such a way that any data more than 60 s before and 100 s after the P-wave arrival can be excluded during the processing. The waveform data with different sampling rates are re-sampled to 10 samples per second (SPS).

2.3.2 ROTATION

Three component seismometers are used to record the particle motions in vertical (Z), North-South (N-S) and East-West (E-W) directions, respectively. This geographical coordinate system is referred as ZNE coordinate system (Fig. 2.4a). In this ZNE coordinate system, the two horizontal components rarely correspond exactly to the horizontally polarized S-wave (SH) and vertically polarized S-wave (SV). As a result, the rotation is essential for the horizontal components. Rotation converts horizontal components of seismogram into its radial (R) and transverse (T) components. The R and T components are aligned along and perpendicular to the great circle connecting epicenter and seismograph. The rotation is done about a vertical axis by an angle equal to back azimuth (BAZ) which converts E-W and N-S components into a geographic coordinate system known as ZRT (vertical, radial and tangential) coordinate system (Fig. 2.4b). After rotation, the P-wave dominates the vertical component - Z, whereas the Ps converted phases and its multiples are recorded on the radial - SV component. Rotation of ZNE system into ZRT system is based on the theoretical BAZ of the recorded event. Thus, the rotation is one of the necessary processing steps to convert horizontal components into their right orientation or coordinate system.
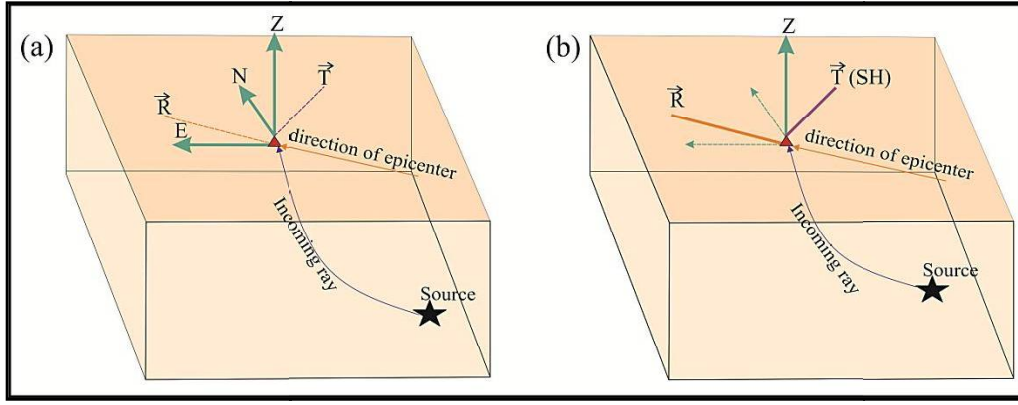
Figure 2.4 Ray diagrams showing the rotation of ZNE to ZRT coordinate system. (a) ZNE coordinate system (b) ZRT coordinate system.

If u is the ground motion due to an earthquake, its components along NS, EW (ZNE coordinate system) as well as radial and transverse (ZRT coordinate system) directions can be denoted by u_N, u_E, u_R and u_T, respectively. The components can be transformed from one coordinate system to the other with the help of the following rotation matrix.

$$\begin{bmatrix} u_R \\ u_T \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} u_N \\ u_E \end{bmatrix} \qquad (2.11)$$

where $\Theta = 3\pi/2 - \xi$ is the rotation angle and ξ is the BAZ at station.

The eqn. 2.11 is used to rotate the horizontal component seismogram into ZRT coordinate system.

2.3.3 FILTERING

The teleseismic waveforms used for RF analysis are filtered with low pass Gaussian filter in order to remove the high frequency components of the wave. The Gaussian filters, $G(\omega)$ is expressed as:

$$G(\omega) = exp(-\omega^2 / (4\,a^2)) \qquad (2.12)$$

The Gaussian filter width parameter "a" controls the frequency content and defines the band width by excluding the high-frequency signals. The filter gain at $\omega = 0$ is unity (a unit-area pulse). The approximate values of pulse width with frequency is listed in Table 2.1 by taking $G(\omega) = 0.1$. Low frequency

· value of a

whereas the high-frequency components are examined by the higher value. (Cassidy, 1992).

Table 2-1 List of Gaussian width parameter with corresponding frequency and approximate pulse width

Gaussian width parameter "a"	Frequency (Hz) at which $G(w) = 0.1$	Approximate Pulse width (s)
10	4.8	0.50
5	2.4	0.75
2.5	1.2	1.00
2.0	1.0	1.25
1.25	0.6	1.50
1.0	0.5	1.67
0.625	0.3	2.10
0.5	0.24	2.36
0.4	0.2	2.04
0.2	0.1	3.73

The vertical resolution of RF depends on the Gaussian width parameter (a) used during computation of RF. Usually, low-frequency waveforms are sufficient to explain the main features of first order discontinuities like Moho and intra crustal layers.

The Gaussian width parameter, $a = 2.0$ has been used in the present study to observe low-frequency content of the signal which corresponds to 0.97 Hz low pass filter. For an average shear wave velocity (Vs) 3.6 km/s, the wavelengths (λ) is= 3.71 km corresponding to frequency 0.97 Hz (following the relation $V=n\lambda$). As the minimum vertical resolvable length is $\lambda/4$ (Sheriff &Geldart, 1995), therefore the layer thickness larger then ~ 1 km is resolvable.

The RFs has also been computed using $a = 5.0$ that corresponds to 2.4 Hz low pass filter. The high-frequency RFs are useful for identifying intra-crustal features.

2.3.4 DECONVOLUTION

The iterative time domain deconvolution method of Ligorriaand Ammon (1999) has been applied in the present study for computation of RRFs and TRFs. The RRFs are useful for understanding underlying earth's structure beneath the recording site and are used for inversion to obtain 1-D shear wave velocity model beneath a recording site. For homogeneous and horizontally stratified isotropic crust, the amplitude of the TRFs should theoretically be zero. The non-zero amplitude of TRFs provides a measure of deviation from this ideal case. The coherent phases present in TRFs may originate due to dipping interfaces, scattering and/or crustal anisotropy (Cassidy, 1992; Savage, 1998).

During deconvolution process, 100 iterations have been carried out for obtaining the best waveform fit. The RFs more than 80 percent waveform fit in the deconvolution process are selected for further analysis. The following parameters have been used for the computation of iterative deconvolution.

(i) Phase delay   20.0

(ii) Gaussian width parameter   2.0 and 5.0

(iii) Minimum error to allowed   0.01

2.3.5 MOVEOUT CORRECTION

In order to increase the strength of the Moho converted phases and suppress noise, it is necessary to align and stack RFs from different epicentral distances at each station. The alignment and constructive summation of conversion phases require that the RFs be equalized in terms of their ray parameter, which is known as moveout correction. A single layer over a half space is taken into account to illustrate the moveout correction, as shown in Figure 2.5.The tp and t_s are the arrival time of P and S waves, respectively. The same ray parameter

is considered for both the phases for simplification.The t_h is the travel time difference betweenthe two rays with planar wave approximation.
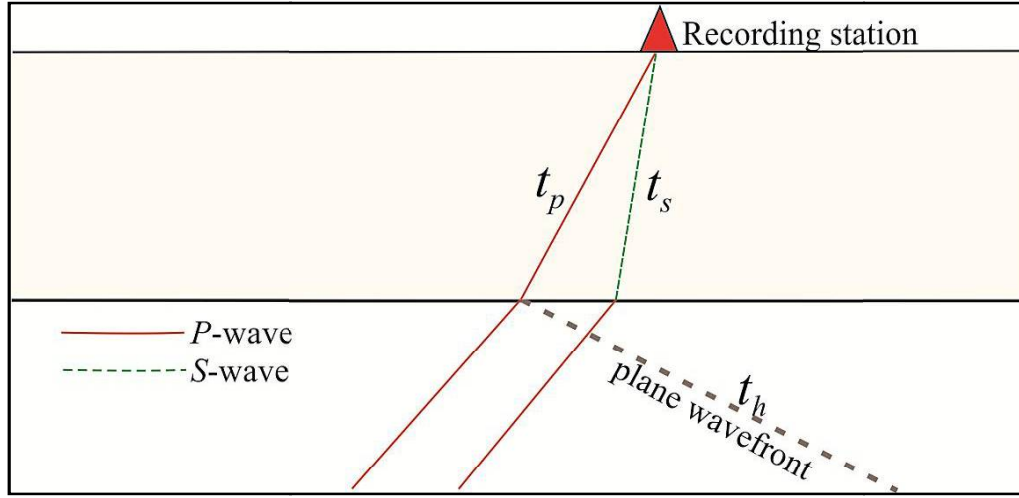

Figure 2.5 Ray paths of direct P-wave (*Pp* phase) and converted *Ps* wave for a simplified layer over a half space.

For teleseismic earthquakes with epicentral distance of more than 30°, the incoming *P* wave can be approximated as plane wave and accordingly *Ps-Pp* delay time can be written as:-

$$t_{ps} - t_p = t_s + t_h - t_p$$

For simplification, the ray parameter is considered same for both *P* and *Ps* phases and following delay-and-sum phase processing technique of Kind and Vinnik, (1988), the delay time of *Ps* phase is represented as:-

$$t_{ps} - t_p = \text{H} \left[(\text{Vs}^{-2} - \text{p}^2)^{1/2} - (\text{Vp}^{-2} - \text{p}^2)^{1/2} \right]$$

Similarly, delay times for crustal multiples (*PpP$_S$* and *PpSs* + *PsPs*) are expressed as:

$$t_{PpPs} - t_p = \text{H} \left[(\text{Vs}^{-2} - \text{p}^2)^{1/2} + (\text{Vp}^{-2} - \text{p}^2)^{1/2} \right] \qquad (2.14)$$

$$t_{PpSs/PsPs} = 2\text{H} (\text{Vs}^{-2} - \text{p}^2)^{1/2} \qquad (2.15)$$

$$t_{PpSs} \quad \text{H} \left[3(\text{Vs}^{-2} - \text{p}^2)^{1/2} + (\text{Vp}^{-2} - \text{p}^2)^{1/2} \right]$$

Different slowness values of crustal multiples and the upper mantle discontinuities are observed when the waves arrive at the recording stations during their conversion at the crust-mantle boundary. Therefore, an adjustment of the time of Ps conversions of all RFs are taken into account pertaining to the arrival time of the Ps conversion at a reference epicentral distance of 67° with ray parameter 0.057 s/km considering IASP91 as reference velocity model (Kennett & Engdahl, 1991). This approach is referred as moveout correction which is similar to normal move-out correction used in exploration seismology. After moveout correction for multiple phases, the direct converted phases (e.g. Ps phases) appear as a straight line and crustal multiples becomes inclined. Stacking of these moveout corrected RFs enhances the Ps conversions while suppressing the multiples considerably. The magnitude of the moveout corrections for shallow depth (e.g. from the Moho or any other discontinuity in the crust) issmall (less than ~0.5 s), while for deep conversions the variations are large.

2.4 STACKING OF RECEIVER FUNCTION

The RFs computed at a station from different events are highly correlated; particularly RFs for earthquakes arriving from a narrow bin of BAZ and Δ are similar in nature. However, some differences in arrival and amplitudes still persist among the individual RFs. The stacking of RFs is important to average out these differences. The stacking of RF for events with similar BAZ and Δ enhances SNR. The stacking procedure increases with SNR. The SNR should go up by a factor of $\sqrt{N}$ if N traces are stacked (Owens et al., 1984). A Standard deviation (SD) of ±1 is calculated from the variance of the stacked data. These bounds are used to check the fit of RFs during modeling. Owens et al.(1984) suggested stacking bounds of 20° BAZ and 10° (for the Δ ranges from 45° to 60°) or 15° (over the Δ ranges from 80° to 100°) in epicentral distance. The processed RFs are stacked with Δ and BAZ bins less than 20° for modeling. At some seismological stations, where BAZ variation is insignificant, the stacking bin is widened in order for the improvement of the SNR.The stacked is RRF obtained from the stacking of individual RRFs is shown in Figure 2.6.
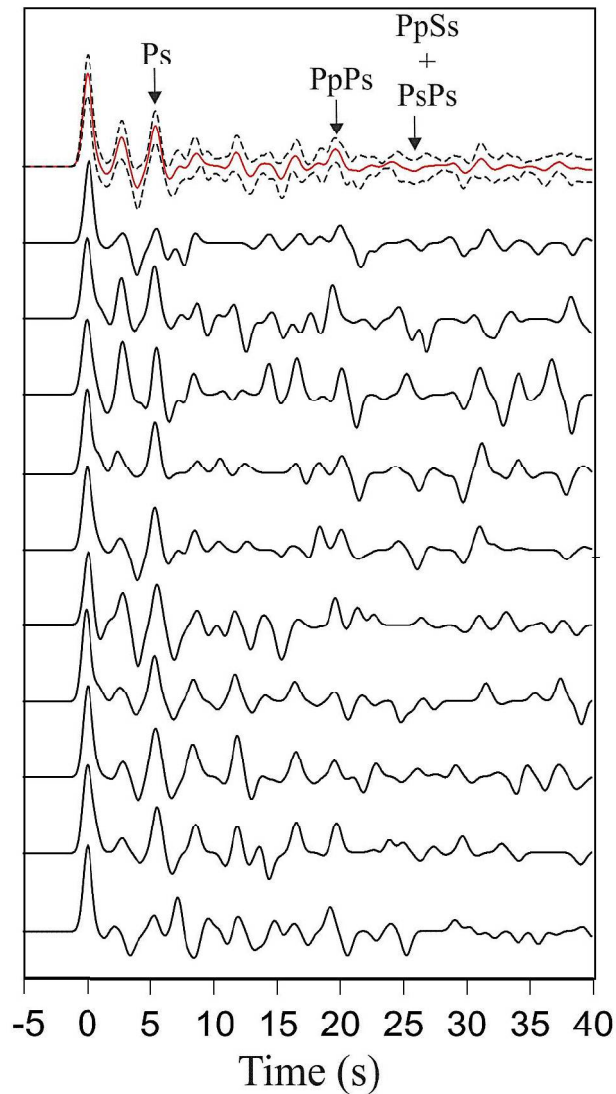
Figure 2.6 Stacking of individual RRFs of teleseismic earthquakes. The individual RFs used for stacking are shown by black waveforms and the stacked RF is shown by red waveform at the top. The waveforms shown by dashes represent ±1SD bounds.

2.4.1 COMMON CONVERSION POINT MIGRATION

Common Conversion Point (CCP) migration of RFs is a useful method to improve the spatial resolution of RF image and uses the idea of Common Midpoint (CMP) stacking and migration in reflection seismology but instead of dealing with reflected waves, it is applied to converted waves. In this method, the time domains RFs are projected into depth domain to obtain geometry of different underlying layers. When a teleseismic *P*-wave interacts

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with any velocity contrasts, the geometry of the ray path at which *P*-wave converts to *S*-wave is defined as Common Conversion Point (Fig. 2.7*a*). In reflection seismology, CMP stacking is a very useful processing method that considers the horizontal reflector model (Fig. 2.7*a*). Similarly, in the RF analysis, the teleseismic*P*-wave is the source and it splits into reflected and refracted *S*-waves after it encounters with the velocity contrast and reaches to the receiver at recording station. Thus, CCP is the conversion point at which the converted waves fall and approaches vertically to the earth's surface (Fig. 2.7*b*).
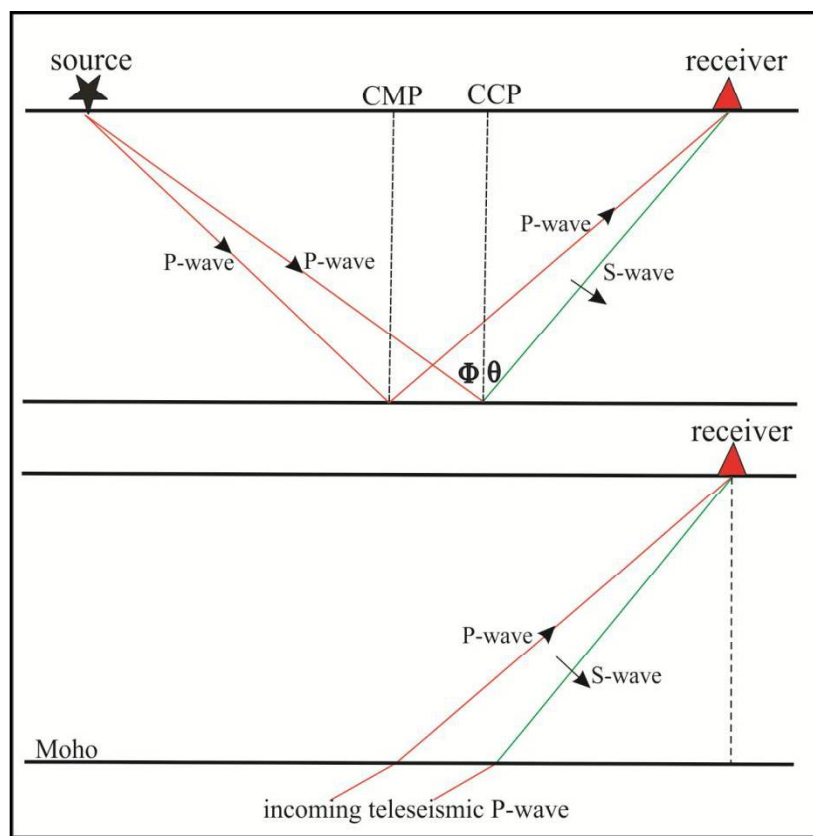

Figure 2.7 Ray diagrams of (a) Common Conversion Point (CCP) and Common Midpoint (CMP) along with (b) propagation of a teleseismic P-wave for a simple earth model.

Various steps like stacking, binning, distance moveout correction and migration are involved to create a CCP image (Dueker& Sheehan, 1997, 1998; Angus et al., 2006). To produce the CCP image, initially recorded seismological data is back-projected with reference to the theoretical ray

paths. Further, ray path of converted waves is binned geographically beneath each station to map the lateral variations in the crustal discontinuities. The ray piercing points are calculated using shear velocity model obtained from standard IASP91 model (Kennett & Engdahl, 1991). After the binning, the time correction is applied on *Ps* converted phases so as to correct for the lateral variations. The binned and time corrected ray paths are summed up to form a stack trace with their corresponding azimuth and ray parameter of each station. Each stacked trace is plotted to produce an image of depth section beneath the seismological study profile. After stacking of time corrected ray paths, the converted phases are move-out corrected in order to remove the variations in source and receiver positions.

Considering the subsurface to be composed of flat horizontal layers for simplicity, the travel time of *Ps* converted phases at various depths is calculated following the relation (Dueker & Sheehan, 1998; Gurrola & Minster, 1998) based on the IASP91 model (Kennett & Engdahl, 1991) with ray parameter, *p* and depth of the discontinuity *d*:

$$T_{pds} = \int_{d}^{0} \left(\sqrt{\frac{1}{Vs(z)^2} - p^2} - \sqrt{\frac{1}{Vp(z)^2} - p^2} \right) dz \qquad (2.17)$$

where, *Vp(z)* and *Vs(z)* are velocities of *P* and *S* waves at a depth *z*, respectively. The accurate values of calculated depths of velocity discontinuities are dependent on the selected reference velocity model. One km variation in the depth to a velocity contrast generated time variation of 0.1 s of the *Ps* converted signal. The SNR and spatial resolution of converted phases are improved and enhanced by CCP technique. Further, post-stack depth migration is performed to produce the true position of reflectors and to generate the correct crustal image. A back-projection migration technique is applied totime and offset corrected bins to solve for a set of point scatterer that generates the true position of all the reflectors (Sheehan et al., 2000). After applying all these steps, each RF of all amplitude in the same bin are stacked to produce the crustal image. The amplitude value of each pixel in the stacked crustal image indicates the impedance contrasts of that particular site beneath

the recording station. Hence, the CCP stacking method improves SNR and spatial resolution of the processed receiver functions and delineates a clear and true crustal image.

2.4.2 H-*k* STACKING METHOD

The H-*k* stacking method developed by Zhu and Kanamori, (2000) is a useful approach in which multiple RFs in the crustal thickness (H)and *Vp/Vs* ratio (*k*) domain are stacked to yield a single robust estimate of H and *k*. The value of *k* is inter-related to Poisson's ratio (σ) by the following relation

$$\sigma = 0.5[1 - (k^2 - 1)^{-1}] \text{ where } k = Vp/Vs \quad (2.18)$$

The Poisson's ratio offers much stronger limitations on the crustal composition than either the compressional or the shear wave velocity alone. The method relies on arrival times and amplitudes of Moho converted *Ps* phase and its crustal multiples (*Ps, PpPs, PpSs + PsPs*). This method is based on the assumption that arrival times and amplitudes of the *Ps* phases and their multiples appearing on radial components of RFs depend on the thickness of the crust underlying the recording site and the *Vp/Vs* ratio or Poisson's ratio.

If t_1 is the differential time between P_S and direct P phase, t_2, the differential time between *PpPS* and P and t_3, the differential time between *PpSS + PsPS* and *P*, thenthe t_1, t_2 and t_3 can be expressed as (following Zhu & Kanamori, 2000)

$$t_1 = H(\sqrt{Vs^{-2} - p^{-2}} - \sqrt{Vp^{-2} - p^{-2}}) \quad (2.19)$$

$$t_2 = H(\sqrt{Vs^{-2} - p^{-2}} + \sqrt{Vp^{-2} - p^{-2}}) \quad (2.20)$$

$$t_3 = 2*H*\sqrt{Vs^{-2} - p^{-2}} \quad (2.21)$$

where H is the thickness of the converting layer and p is the ray parameter. The radial RFs from different BAZ-distance bin are then stacked to obtain travel time of converted and its multiple phases. Based on the above equations (eqns. 2.19, 2.20 and 2.21), it is clear that there is a trade-off between *Vp* and *H*. In order to minimize uncertainty due to this trade-off, the radial RFs of

each crustal multiple are then stacked at each station and the *H-k* domain stack function $S(H, k)$ is defined as:

$$S(H,k) = W1S(t_1) + W2S(t_2) - W3S(t_3) \qquad (2.22)$$

Where $S(t_1)$, $S(t_2)$ and $S(t_3)$ are the stack radial RFs at t_1, t_2 and t_3, respectively. The Wj (j=1, 2, 3) are weighting factors that satisfy $W1 + W2 + W3 = 1$ (Zhu & Kanamori, 2000). The values of weighting parameters are based on SNR of individual phases. To obtain the precise value of crustal thickness and average *Vp/Vs*, it is critical to differentiate different crustal phases and their multiples. The crustal thickness (*H*) and *Vp/Vs* is obtained from radial receiver functions of teleseismic waveforms at each station using H-*k* stacking algorithm. Usually, the *Ps*, *PpPs* and *PpSs+PsPs* phases are weighted as 0.7, 0.2, and 0.1, respectively. However, these values may be different for different waveforms based on clarity and SNR of the individual phases. The optimum value of *H* and *k* is considered as those values for which the stacking amplitude is maximum. The uncertainties of crustal thickness and *Vp/Vs* ratio are calculated using following equations:

$$\sigma^2_H = 2\sigma_s / [\partial^2 s / \partial H^2] \qquad (2.23)$$
$$\sigma^2_k = 2\sigma_s / [\partial^2 s / \partial \kappa^2] \qquad (2.24)$$

where σ_s, is the estimated variance of $S(H,k)$ from stacking (Zhu & Kanamori, 2000). The crustal thickness (H) and Poisson's ratio (σ) are calculated based on Zhu and Kanamori, (2000) method at selected stations showing RF data with a clear recording of *Ps* phase and its crustal multiples.

2.5 INVERSION

RF inversion is one of the complex non-linear geophysical inverse problems. To formulate any geophysical inverse problem, whether it is linear or non-linear, it is essential to have some possible set of finite dimensional model parameters and a defined reference model or model space. Each point in a reference model space describes a number of parameters e.g. physical characteristics of the earth. There are several modeling techniques e.g. time

domain linearized inversion method (Ammon et al., 1990) and Global Optimization technique (Shibutani et al., 1996) that are introduced for inversion of RF data to obtain shear wave velocity structure underneath the recording stations.

To solve the non-uniqueness of RF inversion, Ammon et al. (1990) established linearized time domain waveform inversion technique which is strongly based on the initial model. The inversion technique developed by Hammersley and Handscomb, (1964) is Monte Carlo method that is used for random or probabilistic searching of finite dimensional parameter space. There are several other methods used to solve non-linear inversion problems like Simulated Annealing (SA) technique (Holland, 1975, Kirkpatrick et al., 1983) and Genetic Algorithm (GA) (Stoffa & Sen, 1991). The SA inversion method makes use of control parameters and searches for the best model parameter space, while the GA inversion method uses the initial model space by doing the maximum number of iterations and selects the best-fitted model space which is less dependent on defined constraints of the model. These search methods are mainly used for the optimization of the misfit function in the finite-dimensional parameter space. To overcome the weaknesses of SA and GA inversion techniques, Sambridge, (1999a, b) has developed another non-linear inversion technique called Neighborhood Algorithm (NA) that uses the stochastic sampling in order to search in the multi-dimensional parameter space. In the present study, NA has been utilized for the RF inversion.

2.5.1 NEIGHBORHOOD ALGORITHM

Neighborhood Algorithm is a non-linear inversion method that represents a derivative-free self-adaptive technique that searches the best-accepted models with suitable data fit in a multi-dimensional parameter space (Sambridge, 1999a). It is a random search technique that marks the search of possible sampled model space with good data misfit of parameter space. This method aims to sample the region of high dimensional model parametric space that has models with suitable data fit. It focuses on the different regions of model space and decreases the misfit function through the Neighborhood sampling defined for the inverse problem.

2.5.2 VORONOI CELLS

A Voronoi cell is described as a point 'V' nearest neighbor region in a parametric space or as a distance norm in a model space. The point 'V' is a set of number of points in a particular model space which can be expressed as:-

$$V= \{n_1, n_2............n_{mp}\}, \text{ where } 2 \leq m_p \geq \infty$$

and the Voronoi cell can be represented by

$$V(n_i) = \{x| \parallel x\text{-}a_j \parallel \leq \parallel x\text{-}b_k \parallel \quad \text{for } k \neq j \text{ and } (j, k = 1, 2, 3......,m_p)$$

The randomly distributed points within the finite dimensional parameter model space are searched with the use of geometrical constructed Voronoi cells (Sambridge et al., 1995). Voronoi cells are found to be developed uniquely from the previous samples and inversely proportional to the distribution of sampling points. The distribution of a different number of sample points with their Voronoi cells is shown in Figure 2.8 in a 2D space.
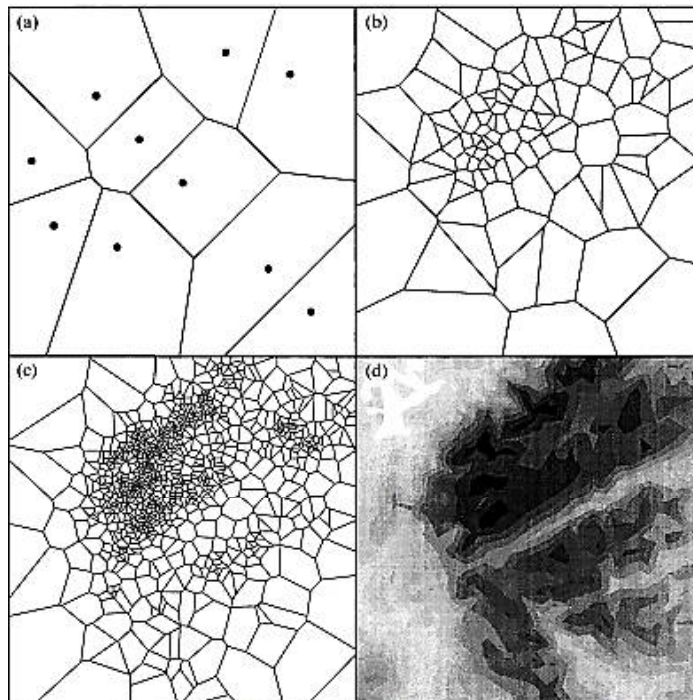

Figure 2.8 Voronoi cells with (a) 10 random sample points, (b) 100 sample points, (c) 1000 sample points and (d) Contours of the test objective function (Sambridge, 1999a).

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Initially, 10 random points (represented as black dots) are distributed in parameterized model space with their drawn Voronoi cells (Fig. 2.8(a)). Further, it generates 10 new random points by doing 10 iterations and total distributed points (10*10 = 100) in a plane is 100 (Fig. 2.8b) and the generated randomly distributed points alter the Voronoi cells locally with their clusters in the model space with the best fit models. In the final step, the 1000 best models are generated by doing 100 iterations 10 times (100*10 = 1000) represented in Figure. 2.8c and Voronoi cells are changed locally with their true misfit values and this increases the resolution of the nearest Neighborhood of multi- parameterized model space.

The foundation of NA inversion technique (Sambridge, 1999a) is very simple and unique and based on two simple principles that are stated below:

1. It requires two tuning parameters, one is numbers of models generated in every iteration (n_s) and selected number of low misfit models (n_r).

2. It requires the best-fitted models to be evaluated for their relative fit to the data as it forms the use of the only rank of misfit function rather than its numerical value.

2.5.3 NEIGHBORHOOD SAMPLING

Nearest Neighborhood Inversion method plays a very significant role in finding the solution of non-linearity and non-uniqueness of RF inversion. The RF inversion is a best data fit procedure that focuses on the estimation of physical properties of earth from the analyzed RF data by providing the predicted initial model space of the crustal properties. This new non-linear inversion method is very simple search behaviour algorithm that requires following steps:

1. Generation of an initial set of n_s models in a defined parameter space (Fig. 2.9).

2. Calculation of misfit function for the recently obtained models n_s and determination of the models n_r with lowest misfit.

3. Generation of new models n_s by searching in the nearest neighbor regions called Voronoi cells.

4. Repeating Step 2 until the misfit between the given initial model space and obtained data becomes satisfactory small.

The NA method has successfully formulated the inversion of RF for identifying the crustal shear wave velocity structure. The earth's structure is divided into six horizontal layers viz. 'sediment', 'basement', 'upper crust', 'middle crust' and 'lower crust' and 'mantle' (Fig. 2.9).
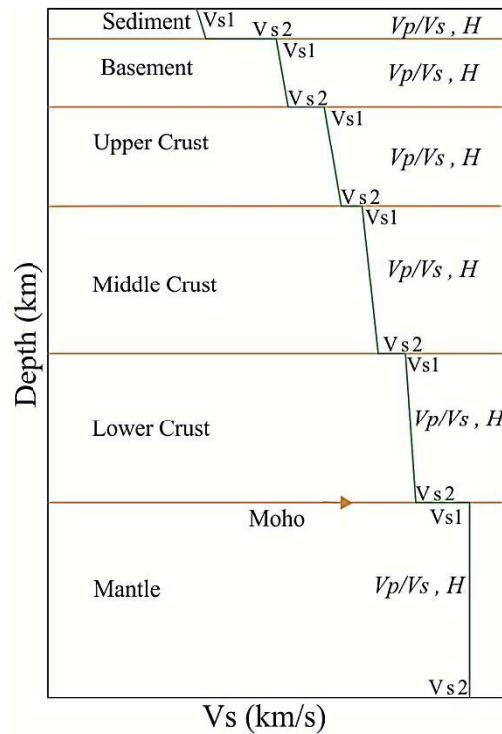

Figure 2.9 Velocity model parameterization for RF inversion indicating six different layers having thickness of layer (H), shear wave velocity at the top and bottom of the layer ($Vs1$ and $Vs2$) and Vp/Vs for each layer. The arrow marks the Moho discontinuity.

This finite dimensional model space is composed of four components on each layer viz. thickness of layer (H), Vs (km/s) of the top layer, Vs of the bottom layer (km/s) and Vp/Vs or Poisson's ratio of the layer. The parameterization uses the large range of earth models to be represented by multi-dimensional parameter space. The gradient of each layer is defined by its maximum and minimum values that are used as input to a layer. On every parameter of each

layer, very loose constraints are applied to obtain the best-fit model for the crustal structure of the earth.