Name: **Enrolment No:**	

UPES

Supplementary Examination, December 2023

Course: Mathematics III (Numerical methods)

Program: B.Tech AE **Semester: III**

Time: 03 hrs

Course Code: MATH2044 **Max. Marks : 100**

Instructions: You must answer all of the questions. You are permitted to utilize a scientific calculator as necessary.

SECTION A
(5Qx4M=20 Marks)

S. No.		Marks	CO
Q 1	Define the order of a numerical method for the solution of the initial value problem $y' = f(x, y), y(x_0) = y_0$. What orders characterize Modified Euler's method and Heun's method?	**4**	**CO3**
Q 2	Write the second order difference approximations for $(i)\ y'(x_i)$ and $(ii)\ y''(x_i)$ based on central differences.	**4**	**CO4**
Q 3	Write out the standard five-point formula for solving (i) Laplace's equation $u_{xx} + u_{yy} = 0$ and (ii) Poisson equation $u_{xx} + u_{yy} = G(x, y)$ with uniform mesh spacing h.	**4**	**CO4**
Q 4	What are the drawbacks of numerical differentiation.	**4**	**CO2**
Q 5	Define an iterative procedure for solving a system of algebraic equations $Ax = b$. What do we mean by convergence of an iterative procedure?	**4**	**CO3**
	SECTION B **(4Qx10M=40 Marks)**		
Q 6	Solve the following system of equations $$26c_1 + 2c_2 + 2c_3 = 12.6$$ $$3c_1 + 27c_2 + c_3 = -14.3$$ $$2c_1 + 3c_2 + 17c_3 = 6.0$$ using Jacobi iteration method. Obtain the results correct to three decimal places.	**10**	**CO3**
Q 7	Consider the initial value problem $y' = x(y + 1), y(0) = 1$. Compute $y(0.2)$ with $h = 0.1$ using Euler's method.	**10**	**CO3**

<table>
<tr><td>Q 8</td><td>Solve the boundary value problem $xy'' + y = 0, y(1) = 1, y(2) = 2$ by second order finite difference method with $h = 0.25$.</td><td>10</td><td>CO4</td></tr>
<tr><td>Q 9</td><td>The velocity of a rocket is given by
$$v(t) = 2000\, ln\left[\frac{14 \times 10^4}{14 \times 10^4 - 2100t}\right] - 9.8t, \qquad 0 \le t \le 30$$
Calculate the acceleration at $t = 16\, s$ using forward and central difference approximations for the first derivative of $v(t)$. In your opinion, which approximation do you believe would offer a more accurate estimate, and what is the reasoning behind your choice? Please utilize a step size of $h = 2\, s$.
OR
Find the missing term in the following table:
<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>$f(x)$</td><td>1</td><td>3</td><td>9</td><td>?</td><td>81</td></tr></table></td><td>10</td><td>CO2</td></tr>
<tr><td></td><td>SECTION C
(2Qx20M=40 Marks)</td><td></td><td></td></tr>
<tr><td>Q 10</td><td>Present an algorithm for executing the Bisection method. Using the bisection method with an initial bracket of [1,5], and after four iterations, determine the iterative approximation of the root for the equation $te^{-t} - 0.3 = 0$.
OR
Provide an algorithm for the implementation of the Newton-Raphson method for finding the root of an algebraic equation. You are working for “DOWN THE TOILET COMPANY” that makes floats for ABC commodes. The floating ball has a specific gravity of 0.6 and has a radius of 5.5 cm. You are asked to find the depth to which the ball is submerged when floating in the water. The equation that gives the depth x in meters to which the ball is submerged underwater is given by
$$x^3 - 0.16\, x^2 + 3.993 \times 10^{-4} = 0.$$
Perform two iteration of the Newton Raphson method to determine the root of the equations that represent the depth x to which the ball is submerged underwater. Start with an initial guess of $x_0 = 0.05\, m$.</td><td>20</td><td>CO1</td></tr>
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Q 11	Solve the following heat conduction equation $$u_{xx} = 32\, u_t, 0 \le x \le 1,$$ taking $h = 0.5$ and $u(x,0) = 0, 0 \le x \le 1, u(0,t) = 0, u(1,t) = t, t > 0$. Use an explicit method with $\lambda = 1/2$. Compute for four time steps.	**20**	**CO4**