Name: **Enrolment No:**	

UPES

Supplementary Examination, December 2023

Course: Mathematics III
Program: B.Tech Civil **Semester: III**
Time: 03 hrs
Course Code: MATH2045 **Max. Marks : 100**

Instructions: You must answer all of the questions. For Question 10, please answer both parts (a) and (b), or alternatively, you may choose to attempt both parts (c) and (d).

SECTION A **(5Qx4M=20 Marks)**			
S. No.		**Marks**	**CO**
Q 1	Find the Laplace transform of $f(t)$ defined as $$f(t) = \begin{cases} 1, 0 < t \le 1 \\ t, 1 < t \le 2 \\ 0, \quad t > 2 \end{cases}$$	**4**	**CO1**
Q 2	Find the inverse transform of $\frac{s^2-3s+4}{s^3}$.	**4**	**CO1**
Q 3	Find the truth table of $\neg\, p \wedge q$.	**4**	**CO2**
Q 4	Consider the set Q of rational numbers with the usual order $\le$. Consider the subset D of Q defined by $D = \{x \mid x \in Q$ and $8 < x^3 < 15\}$. (a) Is D bounded above or below? (b) Does $Sup\ (D)$ or $Inf\ (D)$ exist?	**4**	**CO3**
Q 5	Consider the second order homogeneous recurrence relation $a_n = a_{n-1} + 2\, a_{n-2}$ with initial conditions $a_0 = 2, a_1 = 7$. Find the next three terms of the sequence.	**4**	**CO4**
	SECTION B **(4Qx10M=40 Marks)**		
Q 6	A ball is dropped on a floor from a height of $40\ m$. It is assumed that the ball always rebounds and reaches half of the height from which it falls. If a_n denotes the height it reaches in the n^{th} rebound, then build the numeric function and find a_n. If b_n is the loss in height during the n^{th} rebound, then find b_n and write it in terms of a_n.	**10**	**CO4**
Q 7	Solve the following recurrence relation $a_n - 7a_{n-1} + 10\, a_{n-2} = 0; n \ge 2$, given that $a_0 = 0, a_1 = 41$.	**10**	**CO4**
Q 8	Show that the following argument is a fallacy: $p \to q, \neg\, p \vdash \neg q$.	**10**	**CO2**

Q 9	Suppose $X = \{1,2,6,8,12\}$ is ordered by divisibility and suppose $Y = \{a, b, c, d, e\}$ is isomorphic to X; say, the following function f is a similarity mapping from X onto Y: $$f = \{(1, e), (2, d), (6, b), (8, c), (12, a)\}$$ Draw the Hasse diagram of Y. ***OR*** Let L be a bounded distributive Lattice. Prove that complements are unique, if they exist.	**10**	**CO3**
	SECTION C **(2Qx20M=40 Marks)**		
Q 10	(A) Apply convolution theorem to evaluate the following inverse Laplace transform. $$L^{-1} \frac{s^2}{(s^2 + a^2)\,(s^2 + b^2)}$$ (B) Apply convolution theorem to evaluate the following $$Z^{-1} \frac{z^2}{(z - a)\,(z - b)}$$ ***OR*** (C) Solve by the method of Laplace transforms, the equation $$y''' + 2y'' - y' - 2y = 0$$ given $y(0) = y'(0) = 0$ and $y''(0) = 6$. (D) Find the Z-trnasforms of $\cosh(\frac{n\pi}{2} + \frac{\pi}{4})$.	**20**	**CO1**
Q 11	Consider the third-order homogeneous recurrence relation $a_n = 6\,a_{n-1} - 12\,a_{n-2} + 8a_{n-3}$. (A) Find the general solution. (B) Find the solution with initial conditions $a_0 = 3, a_1 = 4, a_2 = 12$.	**20**	**CO4**