Name: **Enrolment No:**	

UPES

Supplementary Examination, December 2023

Course: Mathematics I **Semester: I**

Program: B. Tech APE UP/APE Gas/CE-RP/Civil/ADE **Time: 03 hrs.**

Course Code: MATH 1026 **Max. Marks: 100**

Instructions: Attempt all questions

SECTION A

(5Qx4M=20Marks)

S. No.		**Marks**	**COs**
Q 1	Find the rank of the following matrix. $$\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$	**4**	**CO1**
Q 2	Find the value of $\frac{\Gamma\left(\frac{9}{2}\right)}{\Gamma\left(\frac{5}{2}\right)}$.	**4**	**CO2**
Q 3	If $z = f(x, y)$ where $x = e^u \cos v$ and $y = e^u \sin v$, show that $y\frac{\partial z}{\partial u} + x\frac{\partial z}{\partial v} = e^{2u}\frac{\partial z}{\partial y}$.	**4**	**CO2**
Q 4	Evaluate $\int_0^1 \int_0^1 \frac{dxdy}{\sqrt{(1-x^2)(1-y^2)}}$	**4**	**CO3**
Q 5	If $\phi = 3x^2y - y^3z^2$; find grad ϕ at the point $(1, -2, -1)$.	**4**	**CO3**

SECTION B

(4Qx10M= 40 Marks)

Q 6	Verify Cayley Hamilton Theorem of the matrix $A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$ and hence find A^{-1}.	**10**	**CO1**

Q 7	Investigate for what values of λ *and* μ the simultaneous equations $$2x + 3y + 5z = 9$$ $$7x + 3y - 2z = 8$$ $$2x + 3y + \lambda z = \mu$$ have (i) no solution (ii) unique solution (iii) infinitely many solutions.	**10**	**CO1**
Q 8	Obtain the half range sine series for the function $\pi x - x^2$ in the interval $(0, \pi)$ up to the first three terms.	**10**	**CO4**
Q 9	Find the Fourier series for the function $f(x) = \begin{cases} \pi x, & 0 \leq x \leq 1 \\ \pi(2 - x), & 1 \leq x \leq 2 \end{cases}$ **OR** If $f(x)$ is a function defined by $f(x) = \begin{cases} x : 0 \leq x \leq \frac{\pi}{2} \\ \pi - x; \frac{\pi}{2} \leq x \leq \pi \end{cases}$, Express $f(x)$ by cosine series.	**10**	**CO4**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10A	If $u = \sin^{-1}\left(\frac{x^3+y^3+z^3}{ax+by+cz}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 2\tan u$	**10**	**CO2**
Q 10B	If $x + y + z = u, y + z = uv, z = uvw$, show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2 v$.	**10**	**CO2**
Q11	Prove that $(y^2 - z^2 + 3yz - 2x)\hat{\imath} + (3xz + 2xy)\hat{\jmath} + (3xy - 2xz + 2z)\hat{k}$ is both solenoidal and irrotational. **OR** A vector field is given by $\vec{F} = (\sin y)\hat{\imath} + x(1 + \cos y)\hat{\jmath}$. Evaluate the line integral over a circular path $x^2 + y^2 = a^2, z = 0$.	**20**	**CO3**

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