Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES, Dehradun
Supplementary Examination, December 2023

Course: Algebra | **Semester: I**
Program: B.Sc. Hons. (Mathematics) | **Time: 03 hrs.**
Course Code: MATH 1040 | **Max. Marks: 100**

Instructions: All questions are compulsory and there are internal choices in Q9 and in Q11.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q1	If $(x+iy)^{1/3}=a+ib$. Prove that $\frac{x}{a}+\frac{y}{b}=4(a^2-b^2)$.	**4**	**CO1**
Q2	Let a, b, and c be integers, where $a\neq 0$. Then prove that, i. if $a\mid b$ and $a\mid c$, then $a\mid(b+c)$ ii. if $a\mid b$ and $b\mid c$, then $a\mid c$	**4**	**CO2**
Q3	The characteristic polynomial of some matrix A is found to be $$p(\lambda)=(\lambda-1)(\lambda-3)^2(\lambda-4)^3$$ a) What is the size of A? b) Is A invertible?	**4**	**CO3**
Q4	Comment on the value of k to have a unique solution for the linear system $x-y=3, 2x-2y=k$.	**4**	**CO3**
Q5	Let T be the linear operator on R^2 defined by $T(x,y)=(x+4y, 2x+3y)$ and β be the standard ordered basis for R^2. Then find the matrix of T with respect to β.	**4**	**CO4**

SECTION B
(4Qx10M= 40 Marks)

S. No.		Marks	CO
Q6	If $x=a+b$, $y=a\omega+b\omega^2$, and $z=a\omega^2+b\omega$, then prove that $$x^3+y^3+z^3=3(a^3+b^3).$$	**10**	**CO1**
Q7	Let R be the relation on the set of ordered pairs of positive integers such that $((a,b),(c,d))\in R$ if and only if $ad=bc$. Show that R is an equivalence relation.	**10**	**CO2**
Q8	Discuss how the rank of A varies with t. $$A=\begin{bmatrix}1 & 1 & t\\ 1 & t & 1\\ t & 1 & 1\end{bmatrix}$$	**10**	**CO3**

Q9	Let A be a 5×7 matrix with rank 4. (a) What is the dimension of the solution space of $AX = 0$? (b) Is $AX = b$ consistent for all vectors b in $\mathbb{R}^5$? Explain. **OR** Check whether the set of vectors $\{1 - 3x + 2x^2,\ 1 + x + 4x^2,\ 1 - 7x\}$ form a basis for P^2 or not.	**10**	**CO4**
SECTION-C **(2Qx20M=40 Marks)**			
Q10	Find the eigen values and eigen vectors of the matrix, $A = \begin{bmatrix} -2 & 2 & 3 \\ -2 & 3 & 2 \\ -4 & 2 & 5 \end{bmatrix}$, and then find the eigenvalues of A^{-1}.	**20**	**CO3**
Q11	Find a basis for the given subspace of $\mathbb{R}^3$ and state its dimension, in each of the following. (a) The plane $3x - 2y + 5z = 0$. (b) The plane $x - y = 0$. (c) The line $x = 2t, y = -t, z = 4t$. (d) All vectors of the form (a, b, c), where $b = a + c$. **OR** Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be a linear transformation defined by the formula $T(x_1, x_2) = (x_1 + 3x_2, x_1 - x_2,\ x_1)$ a) Find the rank of the standard matrix for T. b) Find the nullity of the standard matrix for T.	**20**	**CO4**