Name: **Enrolment No:**	

UPES

Supplementary Examination, December 2023

Course: Engineering Mathematics I **Semester: I**

Program: B. Tech. [APE(UP)+ADE+ Mechatronics+ Mechanical+ Aerospace] **Time : 03 hrs.**

Course Code: MATH 1049 **Max. Marks: 100**

Instructions: All questions are compulsory.

SECTION A

(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Calculate the rank of the matrix $$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$$	**4**	**CO1**
Q 2	Evaluate $\int_0^3 \int_0^1 (x^2 + 3y^2) dx\, dy$	**4**	**CO2**
Q 3	Find a unit vector normal to the surface $x^3 + y^3 + 3xyz = 3$ at the point $(1, 2, -1)$.	**4**	**CO2**
Q 4	Find the divergence and curl of the vector $$\vec{V} = xyz\,\hat{\imath} + 3x^2y\,\hat{\jmath} + (xz^2 - y^2z)\hat{k}.$$	**4**	**CO3**
Q 5	The Fourier series for $f(x)$ in the interval $c < x < c + 2\pi$ is given by $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$ Find the coefficient a_0 for $f(x) = sin^5x$ from $x = -\pi$ to $x = \pi$.	**4**	**CO4**

SECTION B

(4Qx10M= 40 Marks)

Q 6	Using Cayley-Hamilton Theorem find the inverse of $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$.	**10**	**CO1**
Q 7	Change the order of the integration in the integral $\int_0^1 \int_{x^2}^{2-x} xy\, dy\, dx$ and hence evaluate the same.	**10**	**CO2**
Q 8	Show that $\vec{F} = (2xy + z^3)\hat{\imath} + x^2\hat{\jmath} + 3xz^2\hat{k}$ is a conservative force field. Find the scalar potential.	**10**	**CO3**
Q 9	Using Maclaurin's series, expand $\tan x$ up to the term containing x^3.	**10**	**CO4**

	OR Expand $f(x) = x$ as half range (i) $sine$ series in $0 < x < 2$, (ii) $cosine$ series in $0 < x < 2$.		
SECTION-C **(2Qx20M=40 Marks)**			
Q10 A	Evaluate $\iint_S \vec{A}.\hat{n}\, dS$, where $\vec{A} = z\,\hat{\imath} + x\,\hat{\jmath} - 3y^2z\,\hat{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$. **OR** Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point $P(1,2,3)$ in the direction of the line PQ where Q is the point $(5,0,4)$.	**10**	**CO3**
Q10 B	Using Green's theorem, evaluate $\int_C (x^2ydx + x^2dy)$ where C is the boundary described counter clockwise of the triangle with vertices $(0,0)$, $(1,0)$, $(0,1)$. **OR** Calculate the constants a and b so that the surface $ax^2 - byz = (a+2)x$ is orthogonal to the surface $4x^2y + z^3 = 4$ at the point $(1,-1,2)$.	**10**	**CO3**
Q11 A	Obtain the Fourier series of to represent $f(x) = x^2, -\pi < x < \pi$. Sketch the graph of $f(x)$.	**10**	**CO4**
Q11 B	Find the Fourier series to represent the function $f(x)$ given by $f(x) = \begin{cases} x & for \quad 0 \le x \le \pi \\ 2\pi - x & for\ \pi \le x \le 2\pi \end{cases}.$	**10**	**CO4**