Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES

Supplementary Examinations, December 2023

Course: Matrices **Semester: I**

Program: B.Sc. (Hons.) (Physics/Geology/Chemistry) **Time: 03 hrs.**

Course Code: MATH 1029G **Max. Marks : 100**

Instructions: Attempt all the questions. Q9 and Q11 have internal choice.

SECTION A

(5Qx4M=20Marks)

S. No.		Marks	CO
Q1	If the matrix $\begin{bmatrix} x & 2 & x+2 \\ 3 & 5 & 8 \\ x+1 & 7-x & 12 \end{bmatrix}$ is singular, find the value of x.	**4**	**CO1**
Q2	Define the Inverse of a square matrix and hence find the inverse of $A = \begin{bmatrix} 1 & 5 & -2 \\ 3 & -1 & 4 \\ -3 & 6 & -7 \end{bmatrix}$.	**4**	**CO2**
Q3	Examine whether the vectors $x = (3,1,-4), y = (2,2,-3)$ and $z = (0,-4,1)$ are linearly independent or dependent. If dependent, find the relation between them.	**4**	**CO3**
Q4	Show that the transformation $y_1 = x_1 + 2x_2 + 5x_3$, $y_2 = -x_2 + 2x_3$ and $y_3 = 2x_1 + 4x_2 + 11x_3$ is regular and hence find its inverse transformation.	**4**	**CO4**
Q5	Define Block matrix a relevant example.	**4**	**CO5**

SECTION B

(4Qx10M= 40 Marks)

Q6	**(a)** Show that $A = \begin{bmatrix} i & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{bmatrix}$ is skew-Hermitian and also Unitary. **(b)** If A and B are Hermitian, prove that $AB - BA$ is skew-Hermitian.	**10**	**CO1**
Q7	Solve the system $x + 2y + z = 4, 2x - 3y - z = -3, 3x + y + 2z = 3$ using Crout's method.	**10**	**CO3**
Q8	Solve the system of equations $x - y + z = 1$, $-3x + 2y - 3z = -6$ and $2x - 5y + 4z = 5$ using Cramer's rule.	**10**	**CO3**

Q9	Verify the Caley-Hamilton Theorem for $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence find A^{-1}. **OR** Define the minimal polynomial of a matrix. If $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$, find its minimal polynomial.	**10**	**CO4**
SECTION-C **(2Qx20M=40 Marks)**			
Q10	**(a)** A direct-current (d.c) circuit comprises three closed loops. Applying Kirchhoff's laws to the closed loops gives the following equations for the current flow in milliampere: $2I_1 + 3I_2 - 4I_3 = 26$ $I_1 - 5I_2 - 3I_3 = -87$ $-7I_1 + 2I_2 + 6I_3 = 12$ Using Cramer's rule, solve for I_1, I_2 and I_3. **(b)** Test the consistency and hence solve the following set of equations using the concept of rank. $x + 2y - z = 1$ $3x - 2y + 2z = 2$ $7x - 2y + 3z = 5$	**20**	**CO2**
Q11	Diagonalize the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$. **(OR)** Define eigen values and eigen vectors. Find the eigen values and eigen vectors of $A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$.	**20**	**CO4**