Name: **Enrolment No:**	

UPES

End Semester Examination, December 2023

Course: Advanced Numerical Techniques **Semester: VII**

Program: B.Sc. Mathematics by Research **Time : 03 hrs.**

Course Code: MATH 4011 **Max. Marks: 100**

Instructions: Answer all the questions.

SECTION A
(5Qx4M=20Marks)

S. No.		**Marks**	**CO**
Q 1	For the matrix $P = \begin{pmatrix} 3 & -2 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$, one of the eigen values is -2. Find the corresponding eigen vector.	**4**	**CO1**
Q 2	Obtain the Gershgorin circles for the matrix $A = \begin{bmatrix} 2 & 1 & 3 \\ -1 & 4 & 6 \\ 2 & 3 & 1 \end{bmatrix}$.	**4**	**CO1**
Q 3	Discuss the convergence condition of the iteration method for solving the system of nonlinear equations.	**4**	**CO2**
Q 4	Explain Steepest Descent Algorithm.	**4**	**CO2**
Q 5	What is a two-point boundary value problem. Discuss the conditions for the existence of unique solution for a two-point boundary value problem.	**4**	**CO3**

SECTION B
(4Qx10M= 40 Marks)

Q 6	Determine the largest eigen value and the corresponding eigen vector of the matrix $\begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$ using an appropriate technique.	**10**	**CO1**
Q 7	Use Broyden's method to compute $x^{(2)}$ for the nonlinear system $3x_1^2 - x_2^2 = 0$, $3x_1x_2^2 - x_1^3 - 1 = 0$ using $x^{(0)} = (1\ 1)^T$.	**10**	**CO2**
Q 8	Perform two iterations of the steepest descent method to minimize $f(x,y) = x - y + 2x^2 + 2xy + y^2$ starting from the point $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$.	**10**	**CO2**

Q 9	Using finite difference approximations, solve the equation $y'' = x + y$ with the boundary conditions $y(0) = y(1) = 0$ with $h = \frac{1}{4}$. **(OR)** Solve the boundary value problem $y'' + y + 1 = 0\,, y(0) = y(1) = 0$ for $x = 0.5$ by taking $n = 4$.	**10**	**CO3**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	Solve the nonlinear system $x^2 + xy = 10, y + 3xy^2 = 57$ using fixed point iteration technique with initial values $(x_0, y_0) = (1.5, 3.5)$. **(OR)** Perform two iterations of Newton's method for solving the system of nonlinear equations $x^2 + xy + y^2 = 7, x^3 + y^3 = 9$ by considering the initial approximations as $x_0 = 1.5$ and $y_0 = 0.5$.	**20**	**CO2**
Q 11	Apply Linear shooting technique to solve the boundary value problem $y'' = -\frac{2}{x}y' + \frac{2}{x^2}\,y + \frac{\sin(\log x)}{x^2},\ 1 \le x \le 2$ with conditions $y(1) = 1$ and $y(2) = 2$. Perform 2 iterations using step size $h = 0.1$ (Hint: Use Euler's method to solve the IVPs obtained during the procedure).	**20**	**CO3**