Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES

End Semester Examination, December 2023

Course: B.SC (Mathematics by research) **Semester : VII**

Program: Time Series and Forecasting Methods **Time : 03 hrs.**

Course Code: MATH4014P **Max. Marks: 100**

Instructions: All questions are compulsory.

SECTION A

(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Show that the autocovariance function can be written as $\gamma(s,t) = E[(x_s - \mu_s)(x_t - \mu_t)] = E(x_s x_t) - \mu_s \mu_t$, where $E[x_t] = \mu_t$.	**4**	**CO1**
Q 2	Consider the two series $$x_t = w_t$$ $$y_t = w_t - \theta w_{t-1} + u_t$$ where w_t and u_t are independent white noise series with variances σ_w^2 and σ_u^2 respectively, and θ is an unspecified constant. (a) Determine the $\rho_{xy}(h)$ relating x_t and y_t . (b) Show that x_t and y_t are jointly stationary.	**4**	**CO2**
Q 3	Define ARMA(p, q) model and describe how it is related with MA(q) and AR(p).	**4**	**CO2**
Q 4	Derive the spectral density function of AR(2) with $\phi_1 = 1$ and $\phi_2 = -0.9$.	**4**	**CO3**
Q 5	Define Gaussian time series and a linear process. Is every Gaussian time series linear? Justify your answer.	**4**	**CO4**

SECTION B

(4Qx10M= 40 Marks)

Q 6	Consider the time series $x_t = \beta_1 + \beta_2 t + w_t$, where β_1 and β_2 are known constants and w_t is a white noise process with variance σ_w^2. (a) Determine whether x_t is stationary. (b) Show that the process $y_t = x_t - x_{t-1}$is stationary. (c) Show that the mean of the moving average $v_t = \frac{1}{2q+1}\Sigma_{j=-q}^{q} x_{(t-j)}$ is $\beta_1 + \beta_2 t$, and give a simplified expression for the autocovariance function.	**10**	**CO1**
Q 7	Identify the following model as ARMA(p, q) models, and determine whether it is causal and/or invertible: $$x_t = x_{t-1} - 0.50 x_{t-2} + w_t - w_{t-1}.$$	**10**	**CO2**

Q 8	Find the ACF of an AR(2) process and discuss its tendency as $h \to \infty$.	**10**	**CO3**
Q 9	Let $x_t = \delta + x_{t-1} + w_t$, t = 1,2 .... be the random walk with constant drift δ, defined by $w_0 = 0$ where $w_t \sim wn(0, \sigma_w^2)$. Compute the mean of x_t and the autocovariance of the process $\{x_t\}$. Show that $\{\nabla x_t\}$ is stationary and compute its mean and autocovariance function. OR Prove the squared coherence $\rho_{y.x}^2(\omega) = 1$ for all ω when $y_t = \sum_{r=\infty}^{\infty} a_r x_{t-r}$, that is, when x_t and y_t can be related exactly by a linear filter.	**10**	**CO4**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	a. Describe Signal plus noise series. b. Define a weakly stationary time series. c. Define a linear process. d. Define cross covariance. e. Bivariate Normal Distribution	**20**	**CO1**
Q 11	Suppose the first-order autoregressive process $x_t = \phi x_{t-1} + w_t$ has an observation missing at $t = m$, leading to the observations $y_t = A_t x_t$, where $A_t = 1$ for all t, except $t = m$ wherein $A_t = 0$. Assume $x_0 = 0$ with variance $\sigma_w^2/(1-\phi^2)$, where the variance of w_t is σ_w^2. Show the Kalman smoother estimators in this case are $$x_t^n = \begin{cases} \phi y_1 & t = 0, \\ \dfrac{\phi}{1+\phi^2} & t = m \\ \mathrm{y} & \mathrm{t} \neq 0, \mathrm{m} \end{cases}$$ with mean square covariances determined by $$P_T^n = \begin{cases} \sigma_w^2 & t = 0 \\ \sigma_w^2/(1+\phi^2) & t = m, \\ 0 & \mathrm{t} \neq 0, \mathrm{m} \end{cases}$$ OR Consider a process consisting of a linear trend with an additive noise term consisting of independent random variables w_t with zero means and variances σ_w^2, that is $x_t = \beta_0 + \beta_1 t + w_t$, where β_1 and β_2 are known constants. (a) Prove that x_t is nonstationary. (b) Prove that the first difference series $y_t = x_t - x_{t-1}$is stationary by finding its mean and autocovariance function. (c) Repeat part (b) if w_t is replaced by a general stationary process, say y_t, with mean function μ_y and autocovariance function $\gamma_y(h)$.	**20**	**CO2**