Name:

Enrolment No:

UPES

End Semester Examination, December 2023

Course: Theory of Ordinary Differential Equations **Semester: V**
Program: Integrated B.Sc. M.Sc. Mathematics **Time : 03 hrs.**
Course Code: MATH 3047 **Max. Marks: 100**

Instructions: All questions are compulsory.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Solve the following simultaneous differential equations: $\frac{dx}{dt}+\frac{dy}{dt}-2y=2\cos t-7\sin t,$ $\frac{dx}{dt}-\frac{dy}{dt}+2x=4\cos t-3\sin t.$	**4**	**CO1**
Q 2	Show that e^x and e^{-x} are linearly independent solutions of $\frac{d^2y}{dx^2}-y=0$ on any interval.	**4**	**CO2**
Q 3	Express $2x^3+2x^2-x-3$ in terms of Legendre's polynomials.	**4**	**CO3**
Q 4	Obtain the solution of following boundary value problem $x^2y''(x)+xy'(x)-4y(x)=0$, along with the boundary conditions $\lvert y(0)\rvert<\infty, y'(1)=6$.	**4**	**CO4**
Q 5	Find the Fourier cosine transform of $$f(x)=e^{-3x}+2e^{-5x}$$	**4**	**CO5**

SECTION B
(4Qx10M= 40 Marks)

Q 6	Evaluate the general solution of $$(x-1)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=0.$$ Given that $y(x)=x$ as one solution.	**10**	**CO2**
Q 7	Solve in series the differential equation $(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0,$ about the point $x=0$.	**10**	**CO3**

Q 8	Evaluate all the eigen values and corresponding eigen vectors for the problem $y''(x) + \beta y(x) = 0$ with $y'(0) = y'(L) = 0$.	**10**	**CO4**
Q 9	Find the Laplace transform of $$\frac{1 - \sin t}{t^2}$$ **OR** If $f(t) = t^2 e^{-2t} \cos 5t$, calculate Laplace transform of $f(t)$.	**10**	**CO5**
	SECTION-C **(2Qx20M=40 Marks)**		
Q 10	If $J_n(x)$ is the Bessel function of first kind of order *n* then prove that a) $\frac{d}{dx}[xJ_n(x)J_{n+1}(x)] = x[J_n^2(x) - J_{n+1}^2(x)]$. b) $J_4(x) = \left(\frac{48}{x^3} - \frac{8}{x}\right)J_1(x) + \left(1 - \frac{24}{x^2}\right)J_0(x)$.	**20**	**CO3**
Q 11A	Applying Laplace transform solve the following initial value problem $y''(t) + 25y(t) = 10\cos 5t, y(0) = 2, y'(0) = 0.$ **OR** Obtain the inverse Laplace transform of $$\frac{s^2}{(s^2 + a^2)(s^2 + b^2)}$$	**10**	**CO5**
Q 11B	Find Fourier cosine transform of $$f(x) = \begin{cases} x, & 0 < x < \frac{1}{2} \\ 1 - x, & \frac{1}{2} < x < 1 \\ 0 & x > 1 \end{cases}$$ **OR** Evaluate the Fourier transform of the function $$f(x) = \begin{cases} 1 - \lvert x \rvert & \text{if } \lvert x \rvert < 1 \\ 0 & \text{for } \lvert x \rvert > 1 \end{cases}$$	**10**	**CO5**