Name:

Enrolment No:

UPES

End Semester Examination, December 2023

Course: B.Sc. (H) Mathematics/ Int. B. Sc. M. Sc. Mathematics **Semester: V**

Program: FINITE ELEMENT METHODS **Time : 03 hrs.**

Course Code: MATH 3041 **Max. Marks: 100**

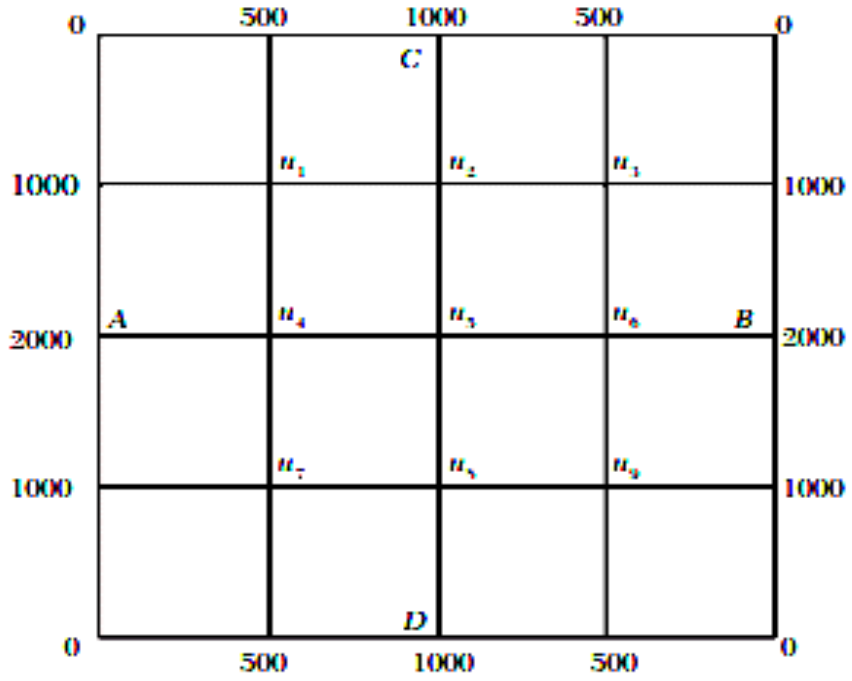
Instructions:Attempt all questions.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	The population of a certain city is given below for various years at equal intervals except for one year which is to be estimated. Year: 1951 1961 1971 1981 1991 Population: (in thousands) 45 43 __ 52 55.	4	CO3
Q 2	Use Picard method to solve the equation $y' = x - y$ subject to the condition $y = 1$ when $x = 0$.	4	CO2
Q 3	Evaluate the interval $I = \int_0^1 \sqrt{1 - x^2}\, dx$ taking $h = 0.25$ by trapezoidal rule.	4	CO4
Q 4	Determine whether the given equation is elliptic or hyperbolic: $(x+1)u_{xx} - 2(x+2)u_{xy} + (x+3)u_{yy} = 0.$	4	CO5
Q 5	Define shape function in finite element method.	4	CO3

SECTION B
(4Qx10M= 40 Marks)

Q 6	Find an approximate solution by method of least squares, of the differential equation $\frac{d^2u}{dx^2} - u = x, \quad 0 \le x \le 1$, with boundary condition $u(0) = u(1) = 0$. Use only two basis functions.	10	CO3
Q 7	The following are the measurements t made on a curve recorded by the oscillograph representing a change of current i due to a change in the conditions of an electric current. t : 1.2 2.0 2.5 3.0 i : 1.36 0.58 0.34 0.20. Using Lagrange's formula, find i at $t = 1.6$.	10	CO1

Q 8	Find an approximate solution by Galerkin's method, of the Poisson equation: $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -1$ defined in domain D where $D = \{x, y \mid -1 \leq x, y \leq 1\}$ and homogenous Dirichlet boundary conditions are prescribed on the boundary, i.e. $u = 0$ on $x = \pm 1$ and $y = \pm 1$. Use only one basis function.	**10**	**CO2**
Q 9	A rocket is launched from the ground. Its acceleration is registered during the first 80 seconds and is given in the table below. Using Simpson's $1/3$ rd rule, find the velocity of the rocket at $t = 80$ seconds. $t\,(sec)$: 0 10 20 30 40 50 60 70 80 $f\,(\frac{cm}{sec^2})$: 30 31.63 33.34 35.47 37.75 40.33 43.25 46.69 50.67. **OR** The speed, v meters per second, of a car, t seconds after it starts, is shown in the following table: t : 0 12 24 36 48 60 72 84 96 108 120 v: 0 3.60 10.08 18.90 21.60 18.54 10.26 5.40 4.50 5.40 9.00 using Simpson's 1/3rd rule, find the distance travelled by the car in 2 minutes.	**10** **10**	**CO4** **CO4**
	SECTION-C **(2Qx20M=40 Marks)**		
Q 10	Solve the elliptic equation $u_{xx} + u_{yy} = 0$ for the following square mesh with boundary values as shown in figure given below:  	**20**	**CO4**

Q 11	Solve the heat conduction problem $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to conditions $u(x,0) = \sin \pi x$, $0 \le x \le 1$, and $u(0,t) = u(1,t) = 0$, using Schmidt method and Crank – Nicolson method, taking $h = 1/3,\ k = 1/36$. **OR** For the boundary value problem $$u'' = \left(\frac{3}{2}\right)u^2, \qquad 0 < x < 1,$$ $$u(0) = 4, \quad u(1) = 1.$$ i) Verify that the variational formulation of the problem is $J[u] = \int_0^1 [(u')^2 + u^3]\, dx$. ii)Use the finite element method, with $h = 1/3$, to derive the elemental equations.	**20**	**CO5**