Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES

End Semester Examination, December 2023

Course: Numerical Methods in Scientific Computing **Semester: V**
Program: BSc (H) Physics **Time : 03 hrs.**
Course Code: PHYS 3026 **Max. Marks: 100**

Instructions:

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q1	Using the data given below, prepare a table up to third divided differences. x : −0.1, 0.0, 0.2, 0.3 $f(x)$: 5.30000, 2.00000, 3.19000, 1.00000	**4**	**CO1**
Q2	Using finite differencing, write the expressions for following derivatives: a) $\frac{\partial^2 f}{\partial x^2}$ b) $\frac{\partial^2 f}{\partial x \partial y}$	**4**	**CO1**
Q3	What is a positive definite matrix? Check if the given matrix is positive definite or not? $A = \begin{bmatrix} 4 & 3 & 0 \\ 3 & 4 & -1 \\ 0 & -1 & 4 \end{bmatrix}$	**4**	**CO1**
Q4	What are Newton Coates formula? Discuss their limitations.	**4**	**CO1**
Q5	Find the eigen values of the following matrix: $\begin{bmatrix} 3 & 2 & -1 \\ 1 & -2 & 3 \\ 2 & 0 & 4 \end{bmatrix}$	**4**	**CO1**

SECTION B
(4Qx10M= 40 Marks)

Q6	A car traveling along a straight road is clocked at a number of points. The data from the observations are given in the following table, where the time is in seconds, the distance is in feet, and the speed is in feet per second.	**10**	**CO3**

	Time: 0, 3, 5, 8, 13; Distance: 0, 225, 383, 623, 993; Speed: 75, 77, 80, 74, 72 Use a Hermite polynomial to predict the position of the car and its speed when $t = 10$ s. **OR** Construct a natural cubic spline to approximate $f(x) = e^{-x}$ by using the vales given by $f(x)$ at $x = 0, 0.25, 0.75,$ and 1.0. Integrate the spline over [0,1], and compare the result to $\int_0^1 e^{-x}dx = 1 - \frac{1}{e}$.		
Q7	A particle of mass m moving through a fluid is subjected to a viscous resistance R, which is a function of the velocity v. The relationship between the resistance R, velocity v, and time t is given by the equation $$t = \int_{v(t_0)}^{v(t)} \frac{m}{R(u)} du$$ Suppose that $R(u) = -v\sqrt{v}$ for a particular fluid, where R is in newtons and v is in m/s. If $m = 10$ kg and $v(t_0) = 10$ m/s, approximate the time required for the particle to slow to $v = 5$ m/s.	**10**	**CO4**
Q8	Use RK2 method to numerically solve the following ODE: $$\frac{dy}{dt} = \frac{2 - 2ty}{t^2 + 1}, \quad 0 \le t \le 1, \; y(0) = 1, \quad \text{with } h = 0.1$$ If the actual solution is given as $$y = \frac{2t + 1}{t^2 + 1}$$ Prepare a table of the error between the approximated and actual values at different times.	**10**	**CO4**
Q9	A projectile of mass $m = 0.11$ kg shot vertically upward with initial velocity $v(0) = 8$ m/s is slowed due to the force of gravity, $F_g = -mg$, and due to air resistance, $F_r = -kv\|v\|$, where $g = 9.8$ m/s^2 and $k = 0.002$ kg/m. The differential equation for the velocity v is given by $$m\frac{dv}{dt} = -mg - kv\|v\|$$ (a) Find the velocity after 0.1, 0.2, …, 1.0 s (b) To the nearest tenth of a second, determine when the projectile reaches its maximum height and begins falling.	**10**	**CO4**
SECTION-C **(2Qx20M=40 Marks)**			
Q10	(**a**) Using 3-step Adam's Bashforth method, solve the following time-dependent ODE: $$\frac{dy}{dt} = \frac{y}{t} - \left(\frac{y}{t}\right)^2, \quad 1 \le t \le 2, \quad y(1) = 1$$ Take $h = 0.2$. The actual solution is $y(t) = \frac{t}{1+\ln t}$ (**10 Marks**)	**20**	**CO4**

	(**b**) Discretize the Poisson Equation given below using finite difference method: $$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=-3 \qquad 1<x<2, \quad 1<y<2$$ $$u(x,1)=2x^2,\ \ u(x,2)=3(x-2)^2, \qquad 1\le x\le 2$$ $$u(1,y)=3y^2, \qquad u(2,y)=2(y-1)^2,\ \ 1\le y\le 2$$ Use $h=k=\frac{1}{2}$. Write the discretized equations in terms of $A\boldsymbol{u}=\boldsymbol{b}$, where A is the coefficient matrix, $\boldsymbol{u}$ is the unknown and $\boldsymbol{b}$ is known vector. Discuss the solution strategy. (**10 Marks**) **OR** (**a**) Using 3-step Adam's Moulton method to solve the following ODE: $$\frac{dy}{dt}=-5y+5t^2+2t, \quad 0\le t\le 1, \quad y(0)=1/3$$ Take $h=0.1$. The actual solution is $y(t)=t^2+\frac{1}{3}e^{-5t}$ (**10 Marks**) (**b**) Discretize the Laplace equation given below using Finite difference method: $$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0 \qquad 1<x<2, \quad 0<y<1$$ $$u(x,0)=2\ln x,\ \ u(x,1)=\ln(x^2+1),\ \ 1\le x\le 2$$ $$u(1,y)=\ln(y^2+1),\ \ u(2,y)=\ln(y^2+4),\ \ 0\le y\le 1$$ Use $h=k=\frac{1}{3}$. Write the discretized equations in terms of $A\boldsymbol{u}=\boldsymbol{b}$, where A is the coefficient matrix, $\boldsymbol{u}$ is the unknown and $\boldsymbol{b}$ is known vector. Discuss the solution strategy. (**10 Marks**)		
Q11	Stress-strain relationships and material properties of a cylinder alternatively subjected to heating and cooling may be studied using the following equation $$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}=\frac{1}{4K}\frac{\partial T}{\partial t}, \quad \frac{1}{2}<r<1,\ \ 0<T$$ where $T=T(r,t)$ is the temperature, r is the radial distance from the center of the cylinder, t is time, and K is a diffusivity coefficient. Find the approximations to $T(r,10)$ for a cylinder with outside radius 1, given the initial and boundary conditions: $$T(1,t)=100+40t, \quad T\left(\frac{1}{2},t\right)=t, \quad 0\le t\le 10$$	**20 Marks**	**CO4**

	$$T(r,0) = 200(r - 0.5), \quad 0.5 \le r \le 1$$ Use BTCS (backward in time and centered in space) with $K = 0.1, k = 0.5$ and $h = \Delta r = 0.1$.		