Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES

End Semester Examination, December 2023

Course: B.Sc. (H) Mathematics/ Int. B. Sc. M. Sc. Mathematics **Semester: V**

Program: FINITE ELEMENT METHODS **Time : 03 hrs.**

Course Code: MATH 3041 **Max. Marks: 100**

Instructions: Attempt all questions.

SECTION A

(5Qx4M=20Marks)

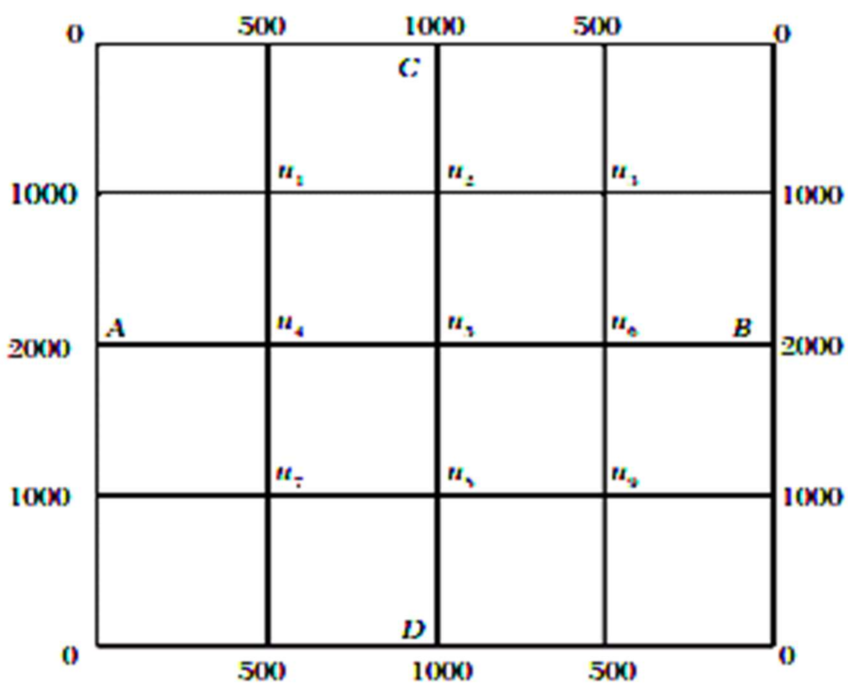
S. No.		**Marks**	**CO**
Q 1	The population of a certain city is given below for various years at equal intervals except for one year which is to be estimated. Year: 1951 1961 1971 1981 1991 Population: 45 43 __ 52 55. (in thousands)	**4**	**CO3**
Q 2	Use Picard method to solve the equation $y' = x - y$ subject to the condition $y = 1$ when $x = 0$.	**4**	**CO2**
Q 3	Evaluate the interval $I = \int_0^1 \sqrt{1-x^2}\, dx$ taking $h = 0.25$ by trapezoidal rule.	**4**	**CO4**
Q 4	Determine whether the given equation is elliptic or hyperbolic: $(x+1)u_{xx} - 2(x+2)u_{xy} + (x+3)u_{yy} = 0.$	**4**	**CO5**
Q 5	Define shape function in finite element method.	**4**	**CO3**

SECTION B

(4Qx10M= 40 Marks)

Q 6	Find an approximate solution by method of least squares, of the differential equation $\frac{d^2u}{dx^2} - u = x, \quad 0 \le x \le 1,$ with boundary condition $u(0) = u(1) = 0$. Use only two basis functions.	**10**	**CO3**
Q 7	The following are the measurements t made on a curve recorded by the oscillograp representing a change of current i due to a change in the conditions of an electric curren t : 1.2 2.0 2.5 3.0 i : 1.36 0.58 0.34 0.20 Using Lagrange's formula, find i at $t = 1.6$.	**10**	**CO1**

<table>
<tr><td>Q 8</td><td>Find an approximate solution by Galerkin’s method, of the Poisson equation: $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=-1$ defined in domain D where $D=\{x,y\mid -1\le x,y\le 1\}$ and homogenous Dirichlet boundary conditions are prescribed on the boundary, i.e. $u=0$ on $x=\pm 1$ and $y=\pm 1$. Use only one basis function.</td><td>10</td><td>CO2</td></tr>
<tr><td>Q 9</td><td>A rocket is launched from the ground. Its acceleration is registered during the first 80 seconds and is given in the table below. Using Simpson’s $1/3$ rd rule, find the velocity of the rocket at $t=80$ seconds.
<table>
<tr><td>$t\ (sec)$:</td><td>0</td><td>10</td><td>20</td><td>30</td><td>40</td><td>50</td><td>60</td><td>70</td><td>80</td></tr>
<tr><td>$f\ (\frac{cm}{sec^2})$:</td><td>30</td><td>31.63</td><td>33.34</td><td>35.47</td><td>37.75</td><td>40.33</td><td>43.25</td><td>46.69</td><td>50.67.</td></tr>
</table>

OR
The speed, v meters per second, of a car, t seconds after it starts, is shown in the following table:
<table>
<tr><td>t :</td><td>0</td><td>12</td><td>24</td><td>36</td><td>48</td><td>60</td><td>72</td><td>84</td><td>96</td><td>108</td><td>120</td></tr>
<tr><td>v :</td><td>0</td><td>3.60</td><td>10.08</td><td>18.90</td><td>21.60</td><td>18.54</td><td>10.26</td><td>5.40</td><td>4.50</td><td>5.40</td><td>9.00</td></tr>
</table>
using Simpson’s $1/3^{rd}$ rule, find the distance travelled by the car in 2 minutes.</td><td>10</td><td>CO4</td></tr>
<tr><td colspan="4">SECTION-C
(2Qx20M=40 Marks)</td></tr>
<tr><td>Q 10</td><td>Solve the elliptic equation $u_{xx}+u_{yy}=0$ for the following square mesh with boundary values as shown in figure given below:

</td><td>20</td><td>CO4</td></tr>
</table>

Q 11	Solve the heat conduction problem $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ subject to conditions $u(x,0)=\sin \pi x$, $0 \le x \le 1$, and $u(0,t)=u(1,t)=0$, using Schmidt method and Crank – Nicolson method, taking $h=1/3$, $k=1/36$. **OR** For the boundary value problem $$u''=\left(\frac{3}{2}\right)u^2, \qquad 0<x<1,$$ $$u(0)=4, \quad u(1)=1.$$ i) Verify that the variational formulation of the problem is $J[u]=\int_0^1[(u')^2+u^3]\,dx$. ii) Use the finite element method, with $h=1/3$, to derive the elemental equations.	**20**	**CO5**