Name:

Enrolment No:

UPES

End Semester Examination, December 2023

Course: B.Sc. (H) Mathematics/ Int. B. Sc. M. Sc. Mathematics **Semester: V**

Program: BOOLEAN ALGEBRA & AUTOMATA THEORY **Time : 03 hrs.**

Course Code: MATH 3040 **Max. Marks: 100**

Instructions: Attempt all questions.

SECTION A

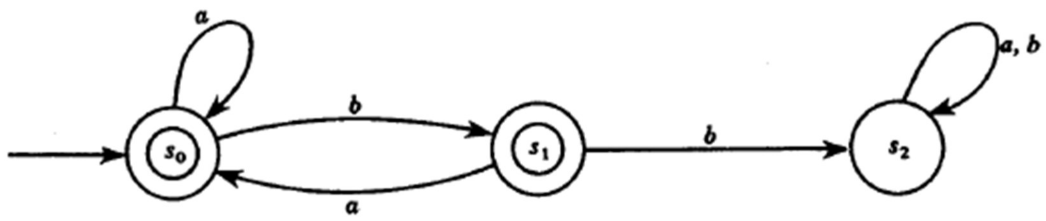
(5Qx4M=20Marks)

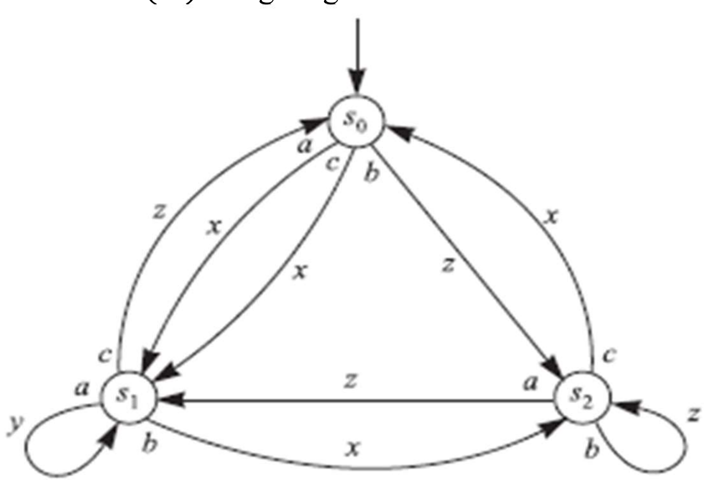
S. No.		Marks	CO
Q 1	Let **N** = {1, 2, 3, . . .} be ordered by divisibility. State whether each of the following subsets of **N** are linearly (totally) ordered. i) $\{24, 2, 6\}$ ii) $\{3, 15, 5\}$.	**4**	**CO2**
Q 2	Define complemented lattice with suitable example.	**4**	**CO1**
Q 3	Define regular language with suitable example.	**4**	**CO3**
Q 4	Find $< m >$ if: i) $m = (4, 0, 3)$ ii) $m = (3, -2, 5)$.	**4**	**CO4**
Q 5	Find the prime implicants and a minimal sum-of-products form for $E = xy + xy'$.	**4**	**CO4**

SECTION B

(4Qx10M= 40 Marks)

Q 6	Find the sum of adjacent products P_1 and P_2 where: i) $P_1 = xyz'$ and $P_2 = xy'z'$. ii) $P_1 = x'yzt$ and $P_2 = x'yz't$. iii) $P_1 = xyz'$ and $P_2 = xyzt$.	**10**	**CO2**
Q 7	Consider the following languages over $A = \{a, b\}$: i) $L_1 = \{a^m b^n \mid m > 0, n > 0\}$; ii) $L_2 = \{b^m a\, b^n \mid m > 0, n > 0\}$. Find a regular expression r over A such that $L_i = L(r)$ for $i = 1, 2$.	**10**	**CO3**
Q 8	Determine whether the automaton M in figure given below accepts the words: $w_1 = ababba$; $w_2 = baab$.	**10**	**CO4**

<table>
<tr><td></td><td>
</td><td></td><td></td></tr>
<tr><td>Q 9</td><td>Find the prime implicants and a minimal sum-of-products form for each of the following complete sum-of-products Boolean expressions:
(a) $E_1 = xyz + xyz' + x'yz' + x'y'z$.
(b) $E_2 = xyz + xyz' + xy'z + x'yz + x'y'z$.
(c) $E_3 = xyz + xyz' + x'yz' + x'y'z' + x'y'z$.
OR

Show that the following are equivalent in a Boolean algebra.
(a) $a + b = b$ (b) $a * b = a$ (c) $a' + b = 1$ (d) $a * b' = 0$.</td><td>10</td><td>CO2</td></tr>
<tr><td colspan="4">SECTION-C
(2Qx20M=40 Marks)</td></tr>
<tr><td>Q 10</td><td>Let M be the finite state machine with input set $A = \{a, b, c\}$, output set $Z = \{x, y, z\}$ and state diagram $D = D(M)$ in figure given below:


i) Construct the state table of M.
ii) Find the output word v if the input is the word: a) $w = a^2c^2b^2cab^3$
b) $w = ca^2b^2ac^2ab$.</td><td>20</td><td>CO5</td></tr>
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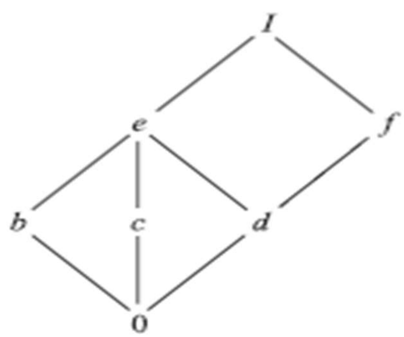
<td>Q 11</td>
<td>
Prerequisites in college is a familiar partial ordering of available classes. We write $A \prec B$ if course A is a prerequisite for course B. Let C be the ordered set consisting of the mathematics courses and their prerequisites appearing in figure given below.
<table>
<tr><th>Class</th><th>Prerequisites</th></tr>
<tr><td>Math 101</td><td>None</td></tr>
<tr><td>Math 201</td><td>Math 101</td></tr>
<tr><td>Math 250</td><td>Math 101</td></tr>
<tr><td>Math 251</td><td>Math 250</td></tr>
<tr><td>Math 340</td><td>Math 201</td></tr>
<tr><td>Math 341</td><td>Math 340</td></tr>
<tr><td>Math 450</td><td>Math 201, Math 250</td></tr>
<tr><td>Math 500</td><td>Math 450, Math 251</td></tr>
</table>
(a) Draw the Hasse diagram for the partial ordering C of these classes.
(b) Find all minimal and maximal elements of C.
(c) Does C have a first element or a last element?
OR
Consider the bounded lattice L in figure given below:


(a) Find the complements, if they exist, of e and f .
(b) Is L distributive?
(c) Describe the isomorphisms of L with itself.
</td>
<td>20</td>
<td>CO2</td>
</tr>
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