Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES

End Semester Examination, December 2023

Course: Mathematical Physics - II **Semester: 3rd**
Program: B.Sc.(H) Phys. **Time : 03 hrs.**
Course Code: PHY 2024 **Max. Marks : 100**

Instructions:

SECTION A
(5Qx4M=20Marks)

S. No.	Answer all the questions	**Marks**	**CO**
Q 1	Show that the order of an element in a group and its inverse is same.	**4**	**CO1**
Q 2	Evaluate the integral $\int_0^{\frac{\pi}{2}}(\sqrt{\tan\theta} + \sqrt{\cot\theta}\,)d\theta$	**4**	**CO2**
Q 3	Find the generating function for the Bessel's function $J_n(x)$.	**4**	**CO2**
Q 4	Derive the series expansion of the error function.	**4**	**CO4**
Q 5	Use the separation of variables to convert the partial differential equation into two ordinary differential equation $$u_{tt} + u_{xt} + u_x = 0$$	**4**	**CO3**

SECTION B
(4Qx10M= 40 Marks)

Q 6	Consider the integral to find the value $\int_0^1 \left(\frac{x}{1-x^3}\right)^{1/2} dx$	**10**	**CO3**
Q 7	Show that the mapping $\varphi: \mathbb{R}^+ \rightarrow \mathbb{R}^+$ defined by $\varphi(x) = \sqrt{x}$ is an automorphism.	**10**	**CO1**
Q 8	If $P_n(x)$ denote the Legendre's polynomial, then show that $\int_{-1}^{1}[P_n(x)]^2\, dx = \frac{2}{2n+1}$.	**10**	**CO2**
Q 9 (a)	Let V be the collection of 2×2 matrices with real entries is a vector space over $\mathbb{R}$, Then show that $W = \{A \in V \mid A^2 = A\}$ is not a subspace of $V(\mathbb{R})$. **OR** If $\{u, v, w\}$ is a linearly independent subset of a vector space $V(\mathbb{R})$, then show that $\{u, u + v, u + v + w\}$ is also linearly independent set.	**5**	**CO1**
Q 9(b)	If V be a vector space over $\mathbb{R}$ with dimension 5, and U and W are two subspaces of V of dimension 3. Then prove that $U \cap W \neq \{0\}$. **OR**	**5**	**CO1**

	Let V be a vector space of collection of all polynomial of degree n with real coefficients. Then establish the basis set for $V(\mathbb{R})$.		
SECTION-C **(2Qx20M=40 Marks)**			
Q10	(a) Establish the relation $\beta(m,n) = \frac{\Gamma(m)\ \Gamma(n)}{\Gamma(m+n)}\quad for\ m,n > 0.$ (12) (b) Prove that $\frac{d}{dx}[erf_c(\alpha x)] = -\frac{2\,\alpha}{\sqrt{\pi}}\ e^{-\alpha^2 x^2}$, all notations have their usual meaning. (8)	**20**	**CO4**
Q 11	Use the separation of variables, to find the solution of the Laplace equation $u_{xx} + u_{yy} = 0$, under the boundary conditions $u(x,0) = 0, \quad (0 < x < 2)$ $u(x,1) = 0, \quad (0 < x < 2)$ $u(0,y) = 0, \quad (0 < y < 1)$ $u(2,y) = a\sin 2\pi y\,, \ (0 < y < 1).$ **OR** A string is stretched and fastened to two points l apart. Motion is started by displacing the string into the form $y = k\,(lx - x^2)$ from which it is released at time $t = 0$. Find the displacement of any point on the string at a distance of x from one end at time t.	**20**	**CO3**