UPES
UNIVERSITY OF TOMORROW

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
Supplementary Examination, December 2023

Course: Engineering Mathematics   Semester: I
Program: B.Tech. SoCS (All Batches)   Time: 03 hrs.
Course Code: MATH 1052   Max. Marks: 100

Instructions: Read all the below mentioned instructions carefully and follow them strictly:
1) Mention Enrolment No. at the top of the question paper.
2) Attempt all the parts of a question at one place only.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Examine the following vectors for linear dependence and find the relation if it exists. $X_1 = (0,0,1,1), X_2 = (1,1,0,0), X_3 = (1,1,1,1)$.	4	CO1
Q 2	Find the nth derivative of $y = e^{3x}(x+2)^3$.	4	CO2
Q 3	Solve $(D+2)^2 y = (e^x x + cosx)$.	4	CO3
Q 4	If $P(1) = P(5)$ in Poisson's distribution, then find the value of its mean.	4	CO4
Q 5	Obtain $\sqrt{12}$, to five places of decimals by Newton Raphson method.	4	CO5

SECTION B
(4Qx10M= 40 Marks)

Q 6	If $y = log(x + \sqrt{x^2 + 1})$, prove that $(x^2 + 1)y_{n+2} + (2n + 1)xy_{n+1} + n^2 y_n = 0$.	10	CO2
Q 7	Solve, by the method of variation of parameters, $\frac{d^2y}{dx^2} - y = e^x$.	10	CO3
Q 8	In a certain distribution, the first four moments about a point are -1.5, 17,-30 and 108. Calculate β_1, β_2 and state whether the distribution is leptokurtic or platykurtic.	10	CO4
Q9	The values of x and y are given as below: 	x	1	3	5	7	9
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y	3	8	12	16	21 | Using Newton's forward interpolation formula, find y at $x = 2$.	10	CO5

	OR		
	Evaluate $\int_0^1 \frac{dx}{(x+1)}$ by using Simpson's 1/3 and 3/8 rule (choose $h = 1/6$). Hence obtain the approximate value of $\log_e 2$.		
	SECTION-C **(2Qx20M=40 Marks)**		
Q 10	**a)** Change the order of integration and hence evaluate $\int_0^a \int_y^a \frac{x \, dxdy}{x^2+y^2}$. **b)** Evaluate $\iint_R (6x^2 - 40y) \, dxdy$, where R is the triangle with vertices $(0, 3), (1, 1)$ and $(5, 3)$. **OR** **a)** Evaluate $\iint_R (4xy - 40y^3) \, dxdy$, where R is the region bounded by $y = \sqrt{x}$ and $y = x^3$. **b)** Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz \, dx \, dy \, dz$.	20	CO2
Q 11	Use Runge – Kutta method of fourth order to find the numerical solution at $x = 0.2$ for $\frac{dy}{dx} = \frac{5x^2-y}{e^{x+y}}$ with $y(0) = 1$. Assume step size $h = 0.1$.	20	CO5