

Name:

Enrolment No:



UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
Supplementary Examination, December 2023

Course: Engineering Mathematics
Program: B.Tech. SoCS (All Batches)
Course Code: MATH 1036

Semester: I
Time: 03 hrs.
Max. Marks: 100

Instructions: Read all the below mentioned instructions carefully and follow them strictly:

- 1) Mention Enrolment No. at the top of the question paper.
- 2) Attempt all the parts of a question at one place only.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and find its inverse.	4	CO1
Q 2	Find the n^{th} derivative of $y = e^x(2x + 7)^3$.	4	CO2
Q 3	Solve $(D - 3)^2y = (e^{3x} + \cos x)$.	4	CO3
Q 4	If $P(1) = P(2)$ in Poisson's distribution, then find the value of its variance.	4	CO4
Q 5	Obtain $\sqrt{12}$, to five places of decimals by Newton Raphson method.	4	CO5

SECTION B
(4Qx10M= 40 Marks)

Q 6	If $y = \sin(m \sin^{-1}x)$, show that $(1 - x^2) y_{n+2} = (2n + 1)xy_{n+1} - (n^2 - m^2)y_n$.	10	CO2												
Q 7	Solve, by the method of variation of parameters, $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 20e^{-2x}$.	10	CO3												
Q 8	In a certain distribution, the first four moments about a point are -1.5, 17, -30 and 108. Calculate β_1, β_2 and state whether the distribution is leptokurtic or platykurtic.	10	CO4												
Q9	The values of x and y are given as below: <table border="1" data-bbox="209 1794 927 1872"><tbody><tr><td>x</td><td>1</td><td>5</td><td>9</td><td>13</td><td>17</td></tr><tr><td>y</td><td>2</td><td>11</td><td>15</td><td>20</td><td>26</td></tr></tbody></table> Using Newton's forward interpolation formula, find y at $x = 3$.	x	1	5	9	13	17	y	2	11	15	20	26	10	CO5
x	1	5	9	13	17										
y	2	11	15	20	26										

OR

Evaluate $\int_0^1 \frac{dx}{(x+1)}$ by using Simpson's 1/3 and trapezoidal rule (choose $h = 1/6$). Hence obtain the approximate value of $\log_e 2$.

SECTION-C
(2Qx20M=40 Marks)

Q 10	<p>a) Change the order of integration and hence evaluate $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dx dy$.</p> <p>b) Find the volume of the solid that lies below the surface given by $z = 16xy + 200$ and lies above the region in the xy-plane bounded by $y = x^2$ and $y = 8 - x^2$.</p> <p>OR</p> <p>a) Evaluate $\iint_R (4xy - 40y^3) dx dy$, where R is the region bounded by $y = \sqrt{x}$ and $y = x^3$.</p> <p>b) Find the volume of the solid enclosed by the planes $4x + 2y + z = 10$, $y = 3x$, $z = 0$, $x = 0$.</p>	20	CO2
Q 11	Use Runge – Kutta method of fourth order to find the numerical solution at $x = 0.2$ for $\frac{dy}{dx} = (x + y) \sin xy$ with $y(0) = 5$. Assume step size $h = 0.1$.	20	CO5