Name:

**Enrolment No:** 



|                          | UNIVERSITY OF TOMORROW  |              |                  |  |  |
|--------------------------|---|--------------|------------------|--|--|
|                          | UPES  |              |                  |  |  |
|                          | End Semester Examination, December 2023   |              |                  |  |  |
| Program                  |   | Semester     | : I              |  |  |
| Course                   | Name: Mathematical Sciences-I   | Time         | : 3 hrs          |  |  |
| Course Code: MATH-1060 N |   | Max. Mar     | Max. Marks : 100 |  |  |
| Nos. of                  | page(s): 2  |              |                  |  |  |
| Instruc                  | tions:  |              |                  |  |  |
| 1.                       | Section A has 5 questions. All questions are compulsory.  |              |                  |  |  |
|                          | Section B has 4 questions. All questions are compulsory. Question 9 has inte  | ernal choice | to               |  |  |
|                          | attempt any one.  |              |                  |  |  |
|                          | Section C has 2 questions. All questions are compulsory. Question 11 has inte   | ernal choice | to attemp        |  |  |
|                          | any one. SECTION A  |              |                  |  |  |
|                          | (5Qx4M=20Marks)   |              |                  |  |  |
| S. No.                   |   | Marks        | СО               |  |  |
|                          |   | Marks        | CO               |  |  |
| Q 1                      | Prove that $\int_0^3 \int_1^2 xy(1+x+y)dy  dx = \int_1^2 \int_0^3 xy(1+x+y)dx  dy$                                    | 4            | CO1              |  |  |
| Q 2                      | Expand log x in powers of $(x - 1)$ .   | 4            | CO1              |  |  |
| Q 3                      | Find the divergence and curl of $\vec{F} = e^{xyz}(xy^2\hat{\imath} + yz^2\hat{\jmath} + zx^2\hat{k})$ at (1,2,3).    | 4            | CO2              |  |  |
| Q 4                      | Solve $(D^2 + 6D + 9)y = 5e^{3x}$ .   | -            |                  |  |  |
| -                        |   | 4            | CO3              |  |  |
| Q 5                      | Determine the solution of $a_n = 4(a_{n-1} - a_{n-2})$ , $n \ge 2$ .  | 4            | CO4              |  |  |
|                          | SECTION B   |              |                  |  |  |
|                          | (4Qx10M= 40 Marks)  |              |                  |  |  |
| Q 6                      | If $u(x, y, z) = (x^2 + y^2 + z^2)^{m/2}$ , then find the value of <i>m</i> which will                                |              |                  |  |  |
|                          | make $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0.$ | 10           | CO1              |  |  |
| ~ -                      |   | 1            |                  |  |  |

|     | make $\frac{\partial u}{\partial x^2} + \frac{\partial u}{\partial y^2} + \frac{\partial u}{\partial z^2} = 0.$   | 10 | COI |
|-----|---|----|-----|
| Q 7 | Evaluate $\int \int \sqrt{4x^2 - y^2}  dx  dy$ over the triangle bounded by $y = 0, y = x$ and $x = 1$ .  | 10 | CO1 |
| Q 8 | If $\vec{A} = (3x^2 + 6y)\hat{\imath} - 14yz\hat{\jmath} + 20xz^2\hat{k}$ , evaluate $\int_C \vec{A} \cdot d\vec{r}$ , where <i>C</i> is the curve $x = t, y = t^2, z = t^3$ from (0,0,0) to (1,1,1). | 10 | CO2 |
| Q 9 | Find the solution of $(D^3 - 3D^2 + 4D - 2)y = e^x + \cos x$<br>OR<br>Solve the following exact differential equation:<br>(ax + hy + g)dx + (hx + by + f)dy = 0                                       | 10 | CO3 |

|      | SECTION-C<br>(2Qx20M=40 Marks)  |    |     |  |  |
|------|---|----|-----|--|--|
| Q 10 | Solve the equation $a_{n+2} - 4a_{n+1} + 4a_n = 2^n$ , $a_0 = 0$ , $a_1 = 1$ .                    | 20 | CO4 |  |  |
| Q 11 | Solve $\frac{d^2y}{dx^2} + y = \sec x$ using method of variation of parameter.                    |    |     |  |  |
|      | OR  | 20 | CO3 |  |  |
|      | Given that $y = e^x$ is a solution, determine the solution of<br>xy'' - (2x - 1)y' + (x - 1)y = 0 |    |     |  |  |