Name: Enrolment No:	

UPES
End Semester Examination, May 2023

Course: Mathematics
Program: B.Tech Biotech/Biomedical/Food technology **Semester: II**
Course Code: MATH1038 **Max. Marks: 100**
Time:3 hours

SECTION A
(5Qx4M=20Marks)

	Instructions: All questions are compulsory.	Marks	CO
Q 1	Evaluate $\lim\limits_{x\to 1}\frac{\sin(\pi x)}{logx}$	4	CO2
Q 2	Reduce the matrix $A=\begin{pmatrix}1 & 2 & -1 & 3\\ 3 & 4 & 0 & -1\\ -1 & 0 & -2 & 7\end{pmatrix}$ to Echelon form, hence find its rank.	4	CO5
Q 3	Verify Rolle's mean value theorem for the function $f(x)=x(x+3)e^{-x/2}$ in the interval $-3\le x\le 0$.	4	CO2
Q 4	Evaluate $\int_{-\infty}^{0}\frac{1}{x^2+4}\,dx$. State whether the improper integral converges or diverges.	4	CO1
Q 5	Apply Taylor's series to expand the function $f(x)=tanx$ in powers of $(x-\frac{\pi}{4})$.	4	CO1

SECTION B
(4Qx10M= 40 Marks)

	Instructions: Section B contains 4 questions. Q9 has internal choice	Marks	CO
Q 6	Examine the consistency of the system and if consistent, solve the equations: $$2x-y-z=2$$ $$x+2y+z=2$$ $$4x-7y-5z=2$$	10	CO5
Q 7	Apply Cauchy root test to test the convergence of the series $$\sum_{n=1}^{\infty}\frac{(n-logn)^n}{(2)^n.n^n}$$	10	CO2
Q 8	Show that $\Gamma\frac{1}{2}=\sqrt{\pi}$	10	CO1

Q 9	Divide 24 into three parts such that the continued product of the first, square of second and cube of third may be maximum. **OR** If $u = log(tanx + tany + tanz)$ then find the value of $$sin2x\frac{\partial u}{\partial x} + sin2y\frac{\partial u}{\partial y} + sin2z\frac{\partial u}{\partial z}$$	**10**	**CO4**
SECTION-C **(2Qx20M=40 Marks)**			
	Instructions: Section C contains 2 questions. Q11 has internal choice	**Marks**	**CO**
Q 10	Find half range sine series of $f(x) = e^{ax}$ in the interval $(0, \pi)$.	**20**	**CO3**
Q 11	Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ and hence find A^{-1}. **OR** Find the Eigen values and Eigen vectors of the matrix, $$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$	**20**	**CO5**