Name:

Enrolment No:

UPES

End Semester Examination, May 2023

Course: Linear Programming and Theory of Games **Semester: VI**
Program: B.Sc. (H) Mathematics **Time : 03 hrs.**
Course Code: MATH 3016 **Max. Marks: 100**

Instructions: Read all the below mentioned instructions carefully and follow them strictly:

1) Mention Roll No. at the top of the question paper.
2) Attempt all the parts of a question at one place only.
3) Attempt all the questions from each section.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Define an extreme point of a LPP. Check whether the following statements are true or false: (***a***) Each extreme point is a boundary point but converse need not be true. (***b***) Extreme point of any convex set may or may not be finite in number.	4	CO1
Q 2	Define degenerate and non-degenerate solution of a LPP.	4	CO1
Q 3	Solve the LPP by using Graphical method Maximize $Z = 8x_1 + 6x_2$ subject to $5x_1 + 4x_2 \leq 80$ $x_1 \leq 12$ $x_2 \leq 15$ $x_1, x_2 \geq 0$	4	CO1
Q 4	Write the dual of the primal problem Maximize $Z = 2x_1 - x_2 + x_3$ subject to $3x_1 + x_2 + x_3 \leq 60$ $x_1 - x_2 + 2x_3 \geq 10$ $x_1 + x_2 - x_3 \leq 20$ $x_1, x_2 \geq 0$ and x_3 is unrestricted in sign.	4	CO2

<table>
<tr>
<td>Q 5</td>
<td>Solve the game whose pay-off matrix is given below
<table>
<tr><td>Player B →
Player A ↓</td><td>B_1</td><td>B_2</td><td>B_3</td></tr>
<tr><td>A_1</td><td>2</td><td>-1</td><td>-2</td></tr>
<tr><td>A_2</td><td>1</td><td>0</td><td>1</td></tr>
<tr><td>A_3</td><td>-2</td><td>-1</td><td>2</td></tr>
</table>
</td>
<td>4</td>
<td>CO5</td>
</tr>
<tr>
<td colspan="4">SECTION B
(4Qx10M= 40 Marks)</td>
</tr>
<tr>
<td>Q 6</td>
<td>Solve an integer linear programming problem using Gomory’s cutting plane method : Maximize $Z = x_1 + 4x_2$
subject to $2x_1 + 4x_2 \leq 7$
$10x_1 + 3x_2 \leq 15$
$x_1, x_2 \geq 0$ and integers.</td>
<td>10</td>
<td>CO1</td>
</tr>
<tr>
<td>Q 7</td>
<td>A salesman is visiting four cities and the cost of visiting the cities as shown in the table below
<table>
<tr><td></td><td>A_1</td><td>A_2</td><td>A_3</td><td>A_4</td></tr>
<tr><td>A_1</td><td>∞</td><td>15</td><td>30</td><td>4</td></tr>
<tr><td>A_2</td><td>6</td><td>∞</td><td>4</td><td>1</td></tr>
<tr><td>A_3</td><td>10</td><td>15</td><td>∞</td><td>16</td></tr>
<tr><td>A_4</td><td>7</td><td>18</td><td>13</td><td>∞</td></tr>
</table>
The salesman can visit each of the cities only once. Determine the optimum sequence he should follow to minimize the total distance travelled. What is the total distance travelled?</td>
<td>10</td>
<td>CO4</td>
</tr>
<tr>
<td>Q 8</td>
<td>Solve the following assignment problem by using Hungarian method
<table>
<tr><td></td><td>I</td><td>II</td><td>III</td><td>IV</td><td>V</td></tr>
<tr><td>A</td><td>2</td><td>3</td><td>5</td><td>1</td><td>4</td></tr>
<tr><td>B</td><td>-1</td><td>1</td><td>3</td><td>6</td><td>2</td></tr>
<tr><td>C</td><td>-2</td><td>4</td><td>3</td><td>5</td><td>0</td></tr>
<tr><td>D</td><td>1</td><td>3</td><td>4</td><td>1</td><td>4</td></tr>
<tr><td>E</td><td>7</td><td>1</td><td>2</td><td>1</td><td>2</td></tr>
</table>
</td>
<td>10</td>
<td>CO4</td>
</tr>
</table>

Q 9

Two companies *A* and *B* are competing in advertising a new product. The marketing research department of company *A* estimates the pay-off matrix. The entries in the following table indicate increased sales in thousands of Rs. for company *A*. Determine the optimal strategies for company A and B and the value of advertising the product.

	Company *B*		
Company A ↓	Television	Newspaper	Radio
Television	4	3	7
Newspaper	2	4	1

OR

Solve the following game by simplex method.

	Player *B*		
Player A	3	-2	4
	-1	4	2
	2	2	6

10

CO5

SECTION-C
(2Qx20M=40 Marks)

Q 10

Solve the linear programming problem

$$\text{Maximize} \quad Z = 3x_1 + x_2 + 4\,x_3$$

$$\text{subject to} \quad 6\,x_1 + 3\,x_2 + 5\,x_3 \le 25$$

$$3\,x_1 + 4\,x_2 + 5\,x_3 \le 20$$

$$x_1, x_2, x_3 \ge 0$$

and test this solution for feasibility and optimality when the

(***a***) objective function is changed to Maximize $Z = 3x_1 + 3\,x_2 + 4\,x_3$.

(***b***) right hand side is changed to $[b_1, b_2]^T = [\,20, 30]^T$.

(***c***) coefficient of x_1 is changed to $[a_{11}, a_{21}]^T = [\,2, 3]^T$.

20

CO2

Q 11

Solve the transportation problem by using Vogel's approximation method

Destinations Sources ↓	D_1	D_2	D_3	D_4	Availabilities
S_1	9	16	15	9	15
S_2	2	1	3	5	25
S_3	6	4	7	3	20
Requirements	10	15	25	10	60

Examine whether this BFS solution is optimal or not. If not then determine the optimal solution.

OR

A company has three production factories F_1, F_2 and F_3 with production capacity of 7, 9 and 18 units per week of product respectively. These units are to be shipped to four warehousesW_1, W_2 , W_3 and W_4 with requirement of 5,8,7 and 14 units per week, respectively. The transportation costs per unit between factories to warehouses are given in the table below. Find the initial basic feasible solution of the following transportation problem by Vogel's Approximation method

Ware house → Factory↓	W_1	W_2	W_3	W_4	Factory Capacity
F_1	21	40	60	18	7
F_2	23	32	42	62	9
F_3	42	18	73	23	18
Warehouse Requirement	5	8	7	14	34

Is the above solution an optimal solution? If not, obtain the optimal solution.

20

CO3