Name: **Enrolment No:**	

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, May 2023

Course: Engineering Mathematics II **Semester: II**
Program: B. Tech. (APE-US, ADE, CHE, AE, APE-GS, ME, MECHATONICS, ECE, ELECTRO-CSE) **Time : 03 hrs.**
Course Code: MATH1051 **Max. Marks: 100**
Instructions: All questions are compulsory.

SECTION A
(5Qx4M=20Marks)

S. No.		**Marks**	**CO**
Q1	Solve the following differential equation $(D^2 - 3D + 2)y = e^{5x}$, where $D \equiv \frac{d}{dx}$	**4**	**CO1**
Q2	If $w = \ln z$ $(z = x + iy)$, find $\frac{dw}{dz}$ and determine where w is non-analytic.	**4**	**CO2**
Q3	Prove that $\int_C \frac{dz}{z-a} = 2\pi i$, where C is the circle $\lvert z - a \rvert = r$	**4**	**CO2**
Q4	Find the nature and location of singularities of the following function $\frac{z - \sin z}{z^2}$	**4**	**CO3**
Q5	Eliminate arbitrary constants a and b from $z = (x-a)^2 + (y-b)^2$ to form the partial differential equation.	**4**	**CO4**

SECTION B
(4Qx10M= 40 Marks)

S. No.		**Marks**	**CO**
Q6	Test whether the equation $(x+y)^2dx - (y^2 - 2xy - x^2)dy = 0$ is exact and hence solve it.	**10**	**CO1**
Q7	Evaluate, using Cauchy's integral formula: $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$, where C is the circle $\lvert z \rvert = 3$	**10**	**CO2**
Q8	Expand the function $f(z) = \sin z$ in a Taylor's series about $z = 0$ and determine the region of convergence.	**10**	**CO3**
Q9	Solve the following partial differential equation $\left(\frac{y^2 z}{x}\right)\frac{\partial z}{\partial x} + (xz)\frac{\partial z}{\partial y} = y^2$ **OR**	**10**	**CO4**

	By using Lagrange's method find the solution of the partial differential equation $y^2\frac{\partial z}{\partial x} - xy\frac{\partial z}{\partial y} = x(z - 2y)$		
SECTION-C **(2Qx20M=40 Marks)**			
Q10A	By integrating around a unit circle, evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{1-2a\cos\theta+a^2} d\theta$, where $a^2 < 1$.	**10**	**CO3**
Q10B	Find Taylor's series expansion of $f(z) = \frac{1}{(z+1)^2}$ about the point $z = -i$.	**10**	**CO3**
Q11	Determine the solution of one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$, $0 < x < L$, under the following conditions boundary conditions: $u(0,t) = u(L,t) = 0$ for all $t > 0$. Initial condition: $u(x,0) = f(x)$. **OR** Using method of separation of variables solve the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}, 0 < x < L,$ subject to the boundary conditions: $u(0,t) = u(L,t) = 0$ for all $t > 0$ and initial conditions: $u(x,0) = f(x)$ and $u_t(x,0) = g(x)$.	**20**	**CO4**