Name: **Enrolment No:**	

UPES

End Semester Examination, May 2023

Course: Calculus **Semester: II**
Program: B. Sc. (Physics, Chemistry, Geology) **Time: 03 hrs.**
Course Code: MATH 1033G **Max. Marks: 100**

Instructions: Read all the below-mentioned instructions carefully and follow them strictly:

1) Mention Roll No. at the top of the question paper.
2) ATTEMPT ALL THE PARTS OF A QUESTION AT ONE PLACE ONLY.

SECTION A

All questions are compulsory **(5Qx4M=20Marks)**

S. No.		**Marks**	**CO**
Q1	Calculate $\lim_{x\to 2}\left(4-\frac{3}{2}x\right)$ using ϵ and δ definition of limit.	**04**	**CO1**
Q2	Apply Leibniz's theorem to prove $$x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$$ when $y = a\cos(\log x) + b\sin(\log x)$.	**04**	**CO2**
Q3	Evaluate the equation of tangent and normal to the curve $$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \text{ at } (a\sec\theta,\ b\tan\theta).$$	**04**	**CO3**
Q4	Analyze the symmetry, origin and point of intersection for the curve $y^2(2a-x) = x^3$.	**04**	**CO4**
Q5	Apply mean value theorem to show that $$\sin x > x - \frac{1}{6}x^3, \text{ if } 0 < x < \frac{\pi}{2}.$$	**04**	**CO6**

SECTION B

All questions are compulsory, and Question 9 has an internal choice

(4Qx10M= 40 Marks)

Q6	Classify the asymptotes of the curve: $$y^3 - x^2y - 2xy^2 + 2x^3 - 7xy + 3y^2 + 2x^2 + 2x + 2y + 1 = 0.$$	**10**	**CO3**

Q7	Trace the curve $x = a\cos^3 t\,, y = b\ \sin^3 t$.	**10**	**CO4**
Q8	Calculate the extrema of the function $$f(x,y) = 4x^2 + 4y^2 + x^3y + yx^3 - xy - 4$$ and the saddle points.	**10**	**CO5**
Q9	Apply Euler's theorem to prove $$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0$$ when $u = \cos^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$. **OR** Evaluate $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ when $u = \frac{x^2y^2}{x+y}$, and hence deduce that $$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\ \frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 6u$$	**10**	**CO6**
SECTION-C **All questions are compulsory, and questions 11(a) and 11(b) have internal choices** **(2Qx20M=40 Marks)**			
Q10(a)	Estimate the length of tangent, subtangent, normal and subnormal to the curve $x = a\,(\theta - \sin\theta), y = a(1-\cos\theta)$ at $\theta = \frac{\pi}{2}$.	**10**	**CO3**
Q10(b)	Discuss the function $f(x) = x^4 - 4x^3$ with respect to increasing and decreasing nature, concavity, point of inflection.	**10**	**CO4**
Q11(a)	Write Taylor's formula for the function $f(x) = \log(1+x), -1 < x < \infty$ about $x = 2$ with Lagrange's form of remainder after 3 terms. **OR** Apply Maclaurin's theorem on $f(x) = (1+x)^4$ to deduce that $(1+x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4$.	**10**	**CO5**
Q11(b)	State and proof Euler's theorem of two variables **OR** If $u = x^y$, then show that $u_{xy} = u_{yx}$.	**10**	**CO6**