Name: Enrolment No:	UPES UNIVERSITY OF TOMORROW

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, May 2022

Course: Mathematics **Semester: II**

Program: B.Tech Food Technology, Biotechnology **Time : 03 hrs.**

Int. B.Tech Food Technology + MBA , Int. B.Tech Biotechnology + MBA

Course Code: MATH 1038 **Max. Marks: 100**

Instructions: Use of calculator is not allowed.

SECTION A

(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Compute the value of Gamma $(\frac{9}{2})$	**4**	**CO1**
Q.2	Verify Lagrange's mean value theorem for the following function $f(x) = x + \frac{1}{x}$ in [1,3]	**4**	**CO2**
Q.3	Test the convergence of the following series whose nth term is $\frac{n^2}{2^n}$, using D'Alembert's ratio test.	**4**	**CO3**
Q.4	If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, Find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$	**4**	**CO4**
Q.5	Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 1 & 3 & 4 \end{bmatrix}$	**4**	**CO5**

SECTION B

(4Qx10M= 40 Marks)

Q .6	Show that $(x^5 - 5x^4 + 5x^3 - 1)$ has a local maximum when $x = 1$, a local minimum when $x = 3$ and neither when $x = 0$.	**10**	**CO1**
Q.7	Verify Rolle's theorem for the function $f(x) = 2x^3 + x^2 - 4x - 2$ in the interval $[-\sqrt{2}, \sqrt{2}\ ]$	**10**	**CO2**
Q.8	Find a Fourier series to represent, $f(x) = \pi - x \ \ , for\ 0 < x < 2\pi$	**10**	**CO3**
Q.9	At any point of the curve, $x = 3cost, y = 3sint, z = 4t,$ Find (i) Tangent Vector (ii) Unit Tangent vector (iii) Normal Vector (iv) Unit normal vector. **OR** Find the maxima and minima of the following function $f(x, y) = 2 + 2x + 2y - x^2 - y^2$	**10**	**CO4**

SECTION-C (2Qx20M=40 Marks)			
Q .10	(a) Find the Taylor's series expansion of $f(x) = 7x^2 - 6x + 1$, about x=2. (b) Show that $\int_0^{\pi/2} \sqrt{sin\theta}\ d\theta\ \times \int_0^{\pi/2} \frac{d\theta}{\sqrt{sin\theta}} = \pi$	**20**	**CO1**
Q.11	(a) Find by elementary row operations the inverse of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ (b) Investigate the values of $\lambda\ and\ \mu$, so that the system : $2x + 3y + 5z = 9, \quad 7x + 3y - 2z = 8, \quad 2x + 3y + \lambda z = \mu$: has (i) a unique solution; (ii) no solution ; (iii) an infinite number of solutions.	**20**	**CO5**