Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, May 2022

Course: Metric Spaces & Complex Analysis **Semester: VI**
Program: B.Sc. (Hons.) Mathematics **Time : 03 hrs.**
Course Code: MATH 3005 **Max. Marks: 100**

Instructions: Attempt all questions.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Give an example of pseudo metric which is not metric.	**4**	**CO1**
Q 2	Give an example of closed set which is a closed sphere.	**4**	**CO2**
Q 3	Find he value of $\int_{-1+i}^{1+i} z\,dz$ along the parabola: $x=t, y=t, -1\le t\le 1$.	**4**	**CO4**
Q 4	Find the residue of $f(z)=\frac{1}{(z-1)(z+2)^2}$ at $z=1$.	**4**	**CO4**
Q 5	Show that $u(x,y)=\frac{1}{2}\log(x^2+y^2)$ is harmonic.	**4**	**CO3**

SECTION B
(4Qx10M= 40 Marks)

Q 6	Find the bilinear transformation which maps the points $z=1, i, -1$ on to the points $w=i, 0, -i$.	**10**	**CO3**
Q 7	If $u-v=(x-y)(x^2+4xy+y^2)$ and $f(z)=u+iv$ is an analytic function of $z=x+iy$, find $f(z)$ in terms of z by Milne Thomson method.	**10**	**CO3**
Q 8	Expand $f(z)=\frac{z}{(z-1)(z-3)}$ in the region: i) $\|z\|<1$ ii) $1<\|z\|<3$.	**10**	**CO4**
Q 9	Show that if $f(z)$ is an analytic function and $f'(z)$ is continuous at each point with in and on the closed curve c, then $\oint_c f(z)\,dz=0$. OR Evaluate $\oint_c \frac{1}{z\sin Z}\,dz$ where C is the triangle with vertices (0, 1), (2, -2), (7, 1).	**10**	**CO4**

	SECTION-C **(2Qx20M=40 Marks)**		
Q 10	a) Prove that continuous image of connected metric space is connected. b) Let (X,d) be a metric space and let E be a connected subset of X such that $E \subset A \cup B$ when A and B are separated subsets of X. Prove that either $E \subset A$ or $E \subset B$.	**20**	**CO2**
Q 11	Using complex variable techniques, evaluate the integral $\int_0^{2\pi} \frac{\sin^2\theta}{a+b\cos\theta} d\theta$. **OR** Using complex variables, evaluate the real integral $\int_0^{\infty} \frac{\cos 3x}{(x^2+1)(x^2+4)} dx$.	**20**	**CO4**