UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, May- 2022

Programme Name: B. Tech. (APE Gas) **Semester: VI**
Course Name : Numerical Methods **Time: 03 hours**
Course Code : MATH3028 **Max. Marks: 100**
Nos. of page(s) : 03

Instructions:

1. Use of scientific calculator is allowed for calculations. Before use, please make sure that it is approved by the invigilator.
2. Any pages used for rough work should be attach along with the answer script.
3. Use of mobile is strictly prohibited.

SECTION A

S. No.		Marks	CO
Q 1	State the difference between Gauss elimination method and Gauss-Siedel method.	**4**	**CO1**
Q 2	In order to find the root of any function, I want you use graphical method. Can you point out two advantages and two disadvantages of the method.	**4**	**CO2**
Q 3	Write the full expression of 3rd order Lagrange interpolating polynomial.	**4**	**CO3**
Q 4	How can you improve the solution obtained by Euler's method to solve ordinary differential equation? Suggest any two method.	**4**	**CO4**
Q 5	What is the difference between Dirichlet and Neumann boundary condition?	**4**	**CO5**

SECTION B

<table>
<tr><td>Q 6</td><td>Use a step size, $h = 3$, and numerically integrate the following using (i) trapezoidal method, and (ii) Simpson's 1/3 rule to:

$$\int_0^6 \frac{1}{1+x^2}\,dx$$

OR

Use appropriate order of Polynomial regression to find the value of $f(x)$ at $\boldsymbol{x = 2}$, from the following data given,

<table>
<tr><th>x</th><th>f(x)</th></tr>
<tr><td>1</td><td>1.648721</td></tr>
<tr><td>3</td><td>4.481689</td></tr>
<tr><td>5</td><td>12.18249</td></tr>
<tr><td>7</td><td>33.11545</td></tr>
</table>
</td><td>10</td><td>CO3</td></tr>
<tr><td>Q 7</td><td>Determine the roots of the function, $f(x) = 4x^3 - 6x^2 + 7x - 2.3$, using false position method to locate the roots. Employ an initial guess of, $x_l = 0$, and $x_u = 1$ and</td><td>10</td><td>CO2</td></tr>
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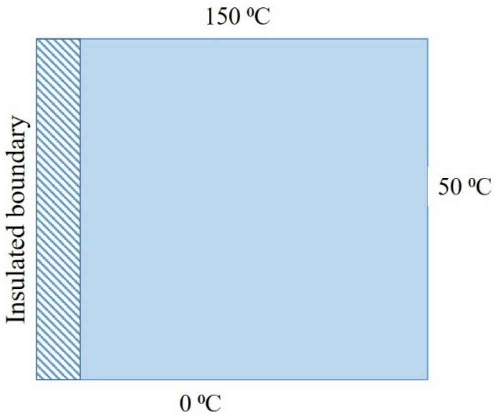
	make **3** iterations and calculate the approximate error, ε_a for each iteration. **OR** Determine the roots of the function, $f(x) = 4x^3 - 6x^2 + 7x - 2.3$, using Newton-Raphson method to locate the roots. Employ an **initial guess** of, $x_o = 0$, and make **3** iterations and calculate the approximate error, ε_a for each iteration.		
Q 8	Use 4th order Runge-Kutta method to numerically solve the following differential equation, $\frac{dy}{dt} = -2y + t^2$ From $t = 0$ to $t = 2$, with a step size (h) of **1**. The initial condition of $y(0) = 1$ is given.	**10**	**CO4**
Q 9	Solve the following heat equation to obtain the temperature distribution of a long, thin rod with a length of 8 cm, from times, $t = 0$ s to $t = 3$ s. The material properties are given as in **Question No. 10** . Use a step size of $\Delta x = 2$ cm, and $\Delta t = 1$ s. At $t = 0$, the temperature of the rod was 5 ^{0}C and the boundary conditions are fixed for all times at $T(0) = 100$ ^{0}C and $T(10) = 50$ ^{0}C. Here, k is the thermal diffusivity. $k\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}$	**10**	**CO5**
SECTION-C			
Q 10	Use finite difference method to obtain the temperature distribution of the square heated plate (Fig. 1). Use a relaxation factor of 1.2. The dimensions of the plate is 8 cm × 8 cm. Use at-least two interior nodes in both horizontal and vertical directions. Note that the material is aluminum with specific heat, $C = 0.2174$ cal/(g · ^{0}C) and density, $\rho = 2.7$ g/cm^3. The thermal conductivity, $k' = 0.49$ cal/(s · cm · ^{0}C), $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$	**20**	**CO5**

			
Q 11	Use **Thomas algorithm** to solve the following simultaneous linear equations: $0.8x_1 - 0.4x_2 = 41$ $-0.4x_1 + 0.8x_2 - 0.4x_3 = 25$ $-0.4x_2 + 0.8x_3 = 105$ Detailed steps should be provided. Check your answers by substituting them into the original equations. **OR** Use **Gauss-Jordan method** to solve the following simultaneous linear equations: $3x_1 + 4x_2 + x_3 = 26$ $x_1 + 2x_2 + 6x_3 = 22$ $6x_1 - x_2 - x_3 = 19$ Detailed steps should be provided. Check your answers by substituting them into the original equations.	**20**	**CO1**