Name:

Enrolment No:

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2022

Course: Mathematics III (Statistical and Numerical Methods) **Semester: III**

Program: B.Tech Mechanical **Time : 03 hrs.**

Course Code: MATH2045 **Max. Marks: 100**

Instructions:

SECTION A

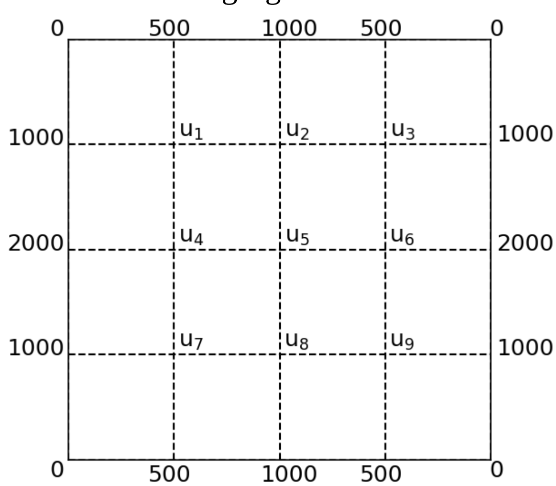
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Let X be a continuous random variable with probability density function $f(x)$ defined as $f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & elsewhere \end{cases}$ Find mean and variance of the distribution. If $2x$ in the definition of the density function is replaced with $3x$, then how your response to the question will change?	**4**	**CO1**
Q 2	A team plays 10 games. The probability of winning a game is 0.4. Find the expected number of wins and variance of winning by the team the probability that the team wins at least 3 games.	**4**	**CO1**
Q 3	Discuss the hypothesis testing for comparing means of two dependent variables.	**4**	**CO2**
Q 4	Differentiate between Newton's forward and backward difference interpolation formulae.	**4**	**CO3**
Q 5	Discuss Picard's method for solving a first order differential equation.	**4**	**CO4**

SECTION B

(4Qx10M= 40 Marks)

Q 6	Define moments of random variables. Explain the significance of first four moments.	**10**	**CO1**
Q 7	A mechanical engineer claims that mean temperature of certain metal in a specific condition in kelvin is 350K. To verify the claim, following temperatures are obtained at randomly selected time at the specific condition of the metal: 340, 356, 332, 362, 318, 344, 386, 402, 322, 360, 362, 354, 340, 372, 338, 375, 364, 355, 324, and 370. Do the data contradict the engineer's claim at 0.05 significance level? (Use $t_{0.025,19} = 2.093$)	**10**	**CO2**

<table>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>Q 8</td><td>Use Simpson's $\frac{1}{3}$rd rule to evaluate $\int_0^1 \frac{1}{1+x}\,dx$ by dividing the interval of integration into 8 equal parts. Use the evaluated value to approximate $\log_e 2$.</td><td>10</td><td>CO3</td></tr>
<tr><td>Q 9</td><td>Solve the boundary value problem $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, under the condition $u(0,t) = u(1,t) = 0$ and $u(x,0) = \sin \pi x$, $0 \le x \le 1$, using Bender-Schmidt method (Take $h = 0.2$, $\alpha = 0.5$)</td><td>10</td><td>CO4</td></tr>
<tr><td colspan="4">SECTION-C
(2Qx20M=40 Marks)</td></tr>
<tr><td>Q 10A</td><td>Compute five iterations of the bisection method to find a root between 2.74 and 2.75 of the function $xlog_{10}x = 1.2$.</td><td>10</td><td>CO3</td></tr>
<tr><td>Q 10B</td><td>Compute five iterations for solving the following system of equations using Gauss-Seidel Iteration method with initial choice as $x = 0$, $y = 0, z = 0$ and $w = 0$
$$x - 0.25y - 0.25z = 50$$
$$-0.25x + y - 0.25w = 50$$
$$-0.25x + z - 0.25w = 25$$
$$-0.25y - 0.25z + w = 25$$</td><td>10</td><td>CO3</td></tr>
<tr><td>Q 11</td><td>Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ using Liebmann's method for the square mesh of the following figure.


OR
Consider a laterally insulated metal bar of length 1 and such that $c^2 = 1$ in the heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$. Suppose that the ends of the bar are kept at temperature $u = 0°C$ and the temperature in the bar at some instant call it $t = 0$ is $f(x) = \sin \pi x$. Applying the Crank-Nicolson method with $h = 0.2$ and $r = 1$, find the temperature $u(x,t)$ in the bar for $0 \le t \le 0.2$.</td><td>20</td><td>CO4</td></tr>
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