UPES
UNIVERSITY OF TOMORROW

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2022

Course: Differential Equations   Semester: III
Program: B. Sc. (Hons.) Physics/Chemistry/Geology &
      Integrated B.Sc.-M.Sc. Physics/Chemistry   Time : 03 hrs.
Course Code: MATH1034G   Max. Marks: 100
Instructions: All questions are compulsory.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Construct a differential equation by the elimination of the arbitrary constants a and b from the equation $ax^2 + by^2 = 1$.	4	CO1
Q 2	Solve the differential equation $y = 3x + \log p$ for p. Here p stands for $\frac{dy}{dx}$.	4	CO2
Q3	Find the complete solution of the following differential equation: $$\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 5y = e^x \cos x$$	4	CO3
Q4	Solve the following simultaneous equations: $$\frac{dy}{dx} + y = z + e^x, \qquad \frac{dz}{dx} + z = y + e^x$$	4	CO4
Q5	Classify the following partial differential equation (a) $2\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + 3\frac{\partial^2 u}{\partial y^2} = 2$ (b) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 u}{\partial y \partial z} + \frac{\partial^2 u}{\partial z \partial y} = 0$	4	CO5

SECTION B
(4Qx10M= 40 Marks)

Q6	Solve the following differential equation: $$(2xy^4 e^y + 2xy^3 + y)dx + (x^2 y^4 e^y - x^2 y^2 - 3x)dy = 0$$	10	CO2
Q7	Find the complete solution of the differential equation: $$x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = \sin(\log x).$$	10	CO3

Q8	Solve the following simultaneous equations: $D^2x - Dy = 2x + 2t,\ Dx + 4Dy = 3y$ Where D stands for $\frac{d}{dt}$	10	CO4
Q9	Find the general solution of the partial differential equation $\{my(x+y) - nz^2\}\frac{\partial z}{\partial x} - \{lx(x+y) - nz^2\}\frac{\partial z}{\partial y} = (lx - my)z$ **OR** Find the integral surface of the partial differential equation $(x-y)\frac{\partial z}{\partial x} + (y-x-z)\frac{\partial z}{\partial y} = z$ through the circle $z = 1, x^2 + y^2 = 1.$	10	CO5

SECTION-C
(2Qx20M=40 Marks)

Q10	**(a)** Use variation of parameter method to solve the following differential equation $$\frac{d^2y}{dx^2} + a^2y = \tan ax$$ **(b)** Find the complete solution of the following differential equation: $$(D^2 + 9)y = x \sin x, D \equiv \frac{d}{dx}$$ **OR** **(a)** Evaluate the solution of the initial value problem $$(D^2 + 1)^2 = 24x \cos x$$ given that $y = Dy = D^2y = 0$ and $D^3y = 12$ when $x = 0$. **(b)** Solve the following differential equation $$(3x + 2)^2\left(\frac{d^2y}{dx^2}\right) + 3(3x + 2)\left(\frac{dy}{dx}\right) - 36y = 3x^2 + 4x + 1$$	10+10	CO3
Q11	**(a)** Using Charpit's method, find a complete integral of $$p^2 - y^2q = y^2 - x^2, p = \frac{\partial z}{\partial x} \text{ and } q = \frac{\partial z}{\partial y}$$ **(b)** Solve the partial differential equation $$(2x^2 + y^2 + z^2 - 2yz - zx - xy)\frac{\partial z}{\partial x}$$ $$+ (x^2 + 2y^2 + z^2 - yz - 2zx - xy)\frac{\partial z}{\partial y}$$ $$= (x^2 + y^2 + 2z^2 - yz - zx - 2xy)$$	10+10	CO5