UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2022

Course: Logic and Sets   **Semester: III**
Program: B.Sc. (H) Mathematics and Int. B.Sc. - M.Sc. Mathematics   **Time : 03 hrs.**
Course Code: MATH 2032K   **Max. Marks: 100**

Instructions: Attempt all the questions. All the questions are compulsory.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	If p be "He is intelligent" and q be "He is tall". Write each statement in symbolic form using p and q. **(i).** He is tall but not intelligent. **(ii).** He is neither tall nor intelligent.	4M	CO1
Q 2	Determine whether the proposition $\sim p \to (p \wedge q)$ is a tautology, contingency or contradiction.	4M	CO2
Q 3	Let $A = \{a, b, c, d, e\}, B = \{a, b, d, f, g\}, C = \{b, c, h\}$, then determine **(i).** $A \setminus (B \cup C)$ **(ii).** $(A \cup B) \setminus C$	4M	CO3
Q 4	Let $B_n = [n, 2n]$, then find (*i*) $B_5 \cap B_8$, (*ii*) $B_1 \cap B_2 \cap B_3$.	4M	CO4
Q 5	Define the partial order relation with an example.	4M	CO5

SECTION B
(4Qx10M= 40 Marks)

Q 6	Show that the proposition $[p \to (q \vee r)] \cong [(p \to q) \vee (p \to r)]$ is equivalent or not?	10M	CO1

Q 7	Determine the validity of the argument
	
	       If I study, then I will not fail in Mathematics.
	       If I do not play football, then I will study.
	       But I failed in Mathematics.
	       ─────────────────────────
	       Thus, I will play football.
		10M	CO1

Q 8	Out of 250 candidates who failed in an examination, it was revealed that 128 failed in Mathematics, 87 in Physics, and 134 in Computer. 31 failed in Mathematics and in Physics, 54 failed in Computer and in Mathematics, 30 failed in the Computer and in Physics. Find how many candidate failed: **(i)** In all the three subjects. **(ii)** In Mathematics but not in Physics. **(iii)** In the Computer but not in Mathematics. **(iv)** In Physics but not in the Computer or in Mathematics. **(v)** In the Computer or in Mathematics, but not in Physics.	**10M**	**CO4**
Q 9	Let $A = \{1, 2, 3, 4, 6, 8, 12, 24, 48\}$ and R be a partial order relation of divisibility on A. Let $B = \{2, 3, 4, 6, 12\}$ be a subset of A, then draw the Hasse's diagram of (A, R) and also determine (*i*) the least upper bound of B and (*ii*) the greatest lower bound of B.	**10M**	**CO5**

SECTION-C (2Qx20M=40 Marks)				

Q 10A	Show that the set of all odd integers is a countable set.	**10M**	**CO3**
Q 10B	State and prove the associative laws for sets.	**10M**	

Q 11A	Determine the principal disjunctive normal form (PDNF) of $(\sim p \rightarrow r) \wedge (p \leftrightarrow q)$.	**10M**	
Q 11B	If $D = \{10, 11, 12, 13, \ldots, 19\}$. Determine the truth value of each of the following statements. **(i).** $(\forall x \in D), x + 14 < 25$ **(ii).** $(\exists x \in D), x + 14 = 20$ **(iii).** $(\forall x \in D), x + 14 \leq 20$ **(iv).** $(\exists x \in D), x + 14 > 25.$	**10M**	
	OR		**CO2**
Q 11A	Determine the principal conjunctive normal form (PCNF) of $(q \wedge p) \vee (\sim q \wedge r)$.	**10M**	
Q 11B	Write the negation of the following statements and also determine their truth values. **(i).** All real numbers are less than 8. **(ii).** For all natural numbers x, if $x > 3$ then $x^2 > 9$.	**10M**	