Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2022

Course: Mathematical Physics I **Semester: I**
Program: BSc (H) Physics **Time : 03 hrs.**
Course Code: PHYS 1011 **Max. Marks: 100**

SECTION A

(5Qx4M=20Marks)

Attempt All Questions. Each Question will carry 4 Marks

S. No.		Marks	CO
Q1	Write a short note on random variables using the coin-tossing example.	4	CO1
Q2	Write down three important properties of Dirac delta function.	4	CO1
Q3	If the xy plane of the Cartesian coordinate system with coordinates $(x, y, z)$is rotated by an angle θ w.r.t. the z axis resulting in a coordinate system with (x', y', z) coordinates, derive the transformation equations relating $(x', y', z) \rightarrow (x, y, z, \theta)$.	4	CO1
Q4	Solve the following differential equation if it is exact: $$xdx + ydy = \frac{a^2(xdy - ydx)}{x^2 + y^2}$$	4	CO3
Q5	Solve the following 1st order linear differential equation: $$\frac{dy}{dx} = \frac{y}{2y\log y + y - x}$$	4	CO2

SECTION B

(4Qx10M= 40 Marks)

Each question will carry 10 marks (10×4 = 40 Marks)

There is an internal choice for Q9.

Q6	Find the complete solution of the following 2nd order linear differential equation: $$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4 = 8x^2e^{2x}\sin 2x$$	10	CO2
Q7	Find the particular solution of the following differential equation using Wronskian method. $$(D^2 - 7D + 10)y = e^{2x}\sin x$$	10	CO2
Q8	Find out whether the differential equation given below	10	CO3

	$$(y-2x^3)dx - x(1-xy)dy = 0$$ Is exact or not? If it is exact, find out the solution. If it is not exact, make it exact and then find out the solution.		
Q9	**(a)** Find the eigen values of the following matrix: (**5 marks**) $$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$ **(b)** Define a Hermitian matrix. Prove that the following matrix is Hermitian: (**5 marks**) $$A = \begin{bmatrix} 1 & 1-i & 2 \\ 1+i & 3 & i \\ 2 & -i & 0 \end{bmatrix}$$ **OR** Find the matrix which diagonalizes the following matrix $$A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$$ Also, write the diagonal matrix.	**10**	**CO1**
SECTION-C **(2Qx20M=40 Marks)** 1. **Each Question carries 20 Marks.** 2. **Attempt two questions. There is an internal choice for Q11.**			
Q10	**(a)** Find the directional derivative of $\vec{\nabla} \cdot \vec{u}$ where $\vec{u} = x^4\hat{\imath} + y^4\hat{\jmath} + z^4\hat{k}$, at the point (1,2,2) in the direction of the outward normal to the sphere $x^2 + y^2 + z^2 = 9$. (**10 marks**) **(b)** A vector field is given by $$\vec{A} = y^2\hat{\imath} + 2xy\hat{\jmath} - z^2\hat{k}$$ Is this field irrotational? If so, find its scalar potential. (**10 marks**)	**20 marks**	**CO4**
Q11	**(a)** State and Discuss Gauss's Divergence theorem. (**5 marks**) **(b)** Evaluate the following surface integral: $$\iint_S \vec{A} \cdot \hat{n}\, ds$$ where $\vec{A} = z\hat{\imath} + x\hat{\jmath} - 3y^2\hat{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ in the first octant between $z = 0$ and $z = 5$. (**15 marks**) **OR** **(a)** State and discuss Stokes' theorem. (**5 marks**) **(b)** Evaluate the following surface integral $$\iint_S \left(yx\hat{\imath} + z\hat{\jmath} + xy^2\hat{k}\right).\overrightarrow{ds}$$ where S is the surface of the sphere $x^2 + y^2 + z^2 = b^2$ in the first octant. (**15 marks**)	**20 marks**	**CO4**