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| **Name:**<br>**Enrolment No:** | UPES UNIVERSITY OF TOMORROW |

**UNIVERSITY OF PETROLEUM AND ENERGY STUDIES**
**End Semester Examination, December 2022**

**Program Name: M.Tech-CSE** — **Semester : I**
**Course Name : Statistical Modelling for Computer Sciences** — **Time : 3 hr**
**Course Code : CSEG 7003** — **Max. Marks : 100**
**Instructions: Attempt all questions. Section B and C has one choice.**

**SECTION A**
**(5Qx4M=20Marks)**

| S. No. | | Marks | CO |
|---|---|---|---|
| Q1. | A single die is rolled six times. Find the probability that the six outcomes are different. (The die is fair and has six faces.) | **4 Marks** | **CO1** |
| Q2. | Mahesh passes through four traffic lights on her way to work, and each light is equally likely to be green or red independent of the others. Find out the PMF, mean, and variance of the number of red lights that Mahesh encounters. | **4 Marks** | **CO2** |
| Q3. | Your probability class has 500 students and each student has probability 1/5 of getting an grade A independent of any other student. Find the mean of X, the number of students that get an grade A.<br>Let X be a random variable that takes values from 0 to 9 with equal probability 1/10.<br>(a) Find the PMF of the random variable Y = X mod (3).<br>(b) Find the PMF of the random variable Y = 5 mod (X + 1). | **4 Marks** | **CO3** |
| Q4. | For $X_1, X_2, . . . , X_n$ independent and identically distributed random variable having mean $\mu$ and variance $\sigma^2$. Prove that for the random variable | **4 Marks** | **CO3** |
| Q5. | State basic characteristic of queuing system. | **4 Marks** | **CO4** |

**SECTION B**
**(4Qx10M= 40 Marks)**

| S. No. | | Marks | CO |
|---|---|---|---|
| Q6. | Each day Bholu eats some rasgullas. On any given day, the number of rasgullas he eats is equally likely to be 1, 2, 3, 4, 5, or 6, independent of what he has eaten in the past. Let X be the number of rasgullas that Bholu eats in 10 days. Find the mean and variance of X. | **10 Marks** | **CO1** |
| Q7. | Suppose that n people throw their hats in a box and then each picks one hat at random. (Each hat can be picked by only one person, and each assignment of hats to persons is equally likely.) Find the expected value of X, the number of people that get back their own hat. | **10 Marks** | **CO2** |

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|---|---|---|---|
| Q8. | Write down the probability mass function (probability density function) and calculate the mean (E[X]), variance (V ar(X)) and standard deviation (std(X)) for the following discrete random variables<br>1. Binomial<br>2. Poisson<br>3. Geometric<br>4. Exponential<br>5. Normal | **10 Marks** | **CO3** |
| Q9. | Briefly, explain about Weak law of Large numbers, strong law of large numbers and the central limits theorem. | **10 Marks** | **CO3** |
| | OR | | |
| | Explain the procedure for testing of Null Hypothesis. | | **CO4** |
| **SECTION-C<br>(2Qx20M=40 Marks)** | | | |
| Q10. | (a) Provide the proof from Binomial to Poisson distribution when $n \to \infty$, $\lambda = np$ and $p$ is very small.<br>$\binom{n}{x} p^x (1-p)^{n-x} \sim e^{-\lambda} \frac{\lambda^x}{x!}$<br>(b) You go to a party with 401 guests. Find the probability that exactly one other guest has the same birthday as you. Calculate this exactly and also approximately by using the Poisson PMF. (For simplicity, exclude birthdays on February 29.) | **20 Marks** | **CO2** |
| Q11. | We toss a coin independently for the 500 times with probability of getting head is p = 0.55.<br>**(a)** Find the probability of getting at most 300 heads out of 500 trials. [Hint: Use the Central Limit Theorem]<br>**(b)** Use the Central Limit Theorem with error correction to find the probability of getting at most 300 heads out of 500 trials. | **20 Marks** | **CO4** |
| | OR | | |
| | Describe application of Markov chain process.<br>Consider the Markov chain shown in Figure. Assume $X_0$=1, and let R be the first time that the chain returns to state 1, i.e., R=min{n≥1:$X_n$=1}. Find E[R\|$X_0$=1]. | **20 Marks** | **CO4** |

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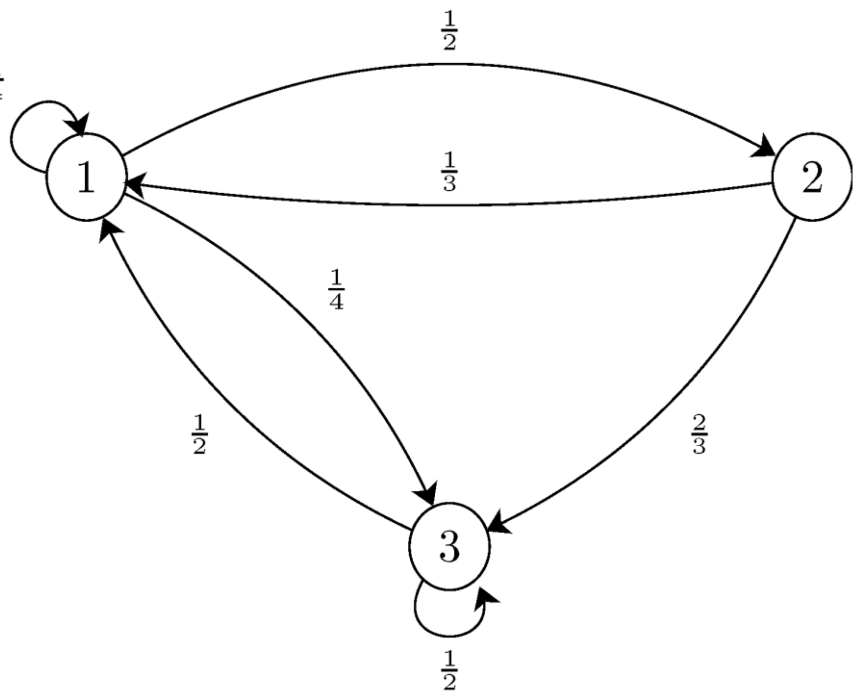
<td>

Figure: A state transition diagram</td>
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