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UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2021

Course: Logic and Sets | **Semester : III**
Program: B.Sc.(H) Mathematics | **Duration : 03 hrs.**
Course Code: MATH 2032 | **Max. Marks: 100**

Instructions: All questions are compulsory

SECTION A
(Scan and upload) **(5Qx 4M = 20 Marks)**

		Marks	COs
Q 1	If p be the statement 'South-West monsoon is very good this year' and q be the statement 'Rivers are rising'. Give the verbal translation of the statement $p \vee \sim q$.	**[4]**	**CO1**
Q 2	Use quantifiers to say that $\sqrt{3}$ is not a rational number.	**[4]**	**CO2**
Q 3	Determine the power set of the set $\{\phi, \{\phi\}\}$.	**[4]**	**CO3**
Q 4	Draw Venn diagram for the following: **(a)** $A^c \cap (B \cup C)$ **(b)** $(A - B) \cap (A - C)$.	**[4]**	**CO4**
Q 5	If $A = \{1,4\}, B = \{2,3\}, C = \{3,5\}$. Prove that $A \times B \neq B \times A$. Also find, $(A \times B) \cap (A \times C)$.	**[4]**	**CO5**

SECTION B
(Scan and upload) **(4Qx10M = 40 Marks)**

Q 1	A survey of 500 TV watchers produced the following information: 285 watch football, 195 watch hockey, 115 watch basketball, 45 watch both football and basketball, 70 watch both football and hockey, 50 watch both hockey and basketball and 50 do not watch any of the three kind of games. **(a)** How many people in the survey watch all three kind of games? **(b)** How many people watch exactly one of the sports?	**[10]**	**CO4**
Q 2	**(a)** Prove that $A \oplus B$ or $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$ **(b)** If $A = \{3,4,2\}, B = \{3,4,5,6\}, C = \{2,4,6,8\}$ then verify that $A \cap (B - C) = (A \cap B) - (A \cap C)$	**[10]**	**CO3**
Q 3	If p: Your car is out of gas. q: You can't drive your car. Write the following propositions using p and q and logical connectives. **(a)** Your car is not out of gas. **(b)** You can't drive your car if it is out of gas. **(c)** Your car is not out of gas if you can drive it. **(d)** If you can't drive your car then it is out of gas. **(e)** If you can't drive your car then your car is not out of gas.	**[10]**	**CO1**

Q 4	Construct truth table to determine whether the following is a tautology, a contingency or a fallacy: $[(p \to q) \wedge (q \to r)] \to (p \to r)$ **OR** Given the following open statements: $p(x): x > 0$, $q(x): x$ is odd, $r(x): x$ is a perfect square, $s(x): x$ is divisible by 3, $t(x): x$ is divisible by 2. Write the following statements in symbolic form **(a)** There exist a positive integer that is odd. **(b)** If x is odd then x is not divisible by 2. **(c)** No odd integer is divisible by 2. **(d)** If x is odd and x is perfect square then x is divisible by 3. **(e)** At least one integer is odd.	**[10]**	**CO2**
SECTION-C **(Scan and upload)** **(2Qx 20M= 40 Marks)**			
Q 1A	If R be a relation in the set of integers Z defined by $R = \{(x, y): x \epsilon \mathrm{Z}, y \epsilon \mathrm{Z},\ (\mathrm{x} - \mathrm{y}) \text{ is divisible by } 6\}$ Then prove that R is an equivalence relation.	**[10]**	**CO5**
Q 1B	If Z be the set of integers. Let aRb if $a = b^r$ for some positive integer r. Show that R is partial order relation on Z.	**[10]**	**CO5**
Q 2A	Write each of the following propositions in the form "p if and only if q" **(a)** If it is hot outside you drink a lot of water, and if you drink a lot of water it is hot outside. **(b)** For a program to be readable it is necessary and sufficient that it is well structured. **(c)** I like fruits only if they are fresh, and fruits are fresh only if I like them. **(d)** If you eat too much sweets your teeth will decay, and conversely. **(e)** The store is closed on exactly those days when I want to shop there. **OR** Consider the following: p: you take a course in Discrete Mathematics. q: You understand logic. r: You get an A on the final exam. Write in simple sentences the meaning of the following: **(a)** $q \to r$ **(b)** $\sim p \to \sim q$ **(c)** $(p \wedge q) \to r$ **(d)** $(p \wedge \sim q) \to \sim r$ **(e)** $\sim (\sim r)$	**[10]**	**CO1**
Q 2B	Test the validity of the following arguments. 'If my brother passes the examination of the Institute of Chartered Accountants in the first attempt, I will give him a valuable prize. Either he passes the examination or I was out of station. I did not give my brother a prize this time although he passed the examination. Therefore, I was out of station.	**[10]**	**CO2**

	OR Give reasons in support of your answers decide if the two compound statements given below are equivalent statements: **(a)** If Ram is 21 years old, then he has right to vote. **(b)** Either Ram is not 21 years old or he has right to vote.		