UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2021

Course	**: Design and Analysis of Algorithms**	**Semester**	**: III**
Program	**: B.Tech. in CSE with all specializations**	**Duration**	**: 03 hrs.**
Course Code	**: CSEG2021**	**Max. Marks**	**: 100**

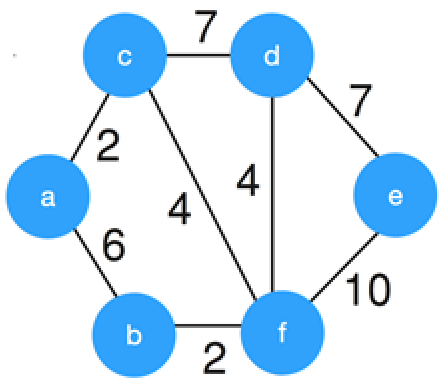
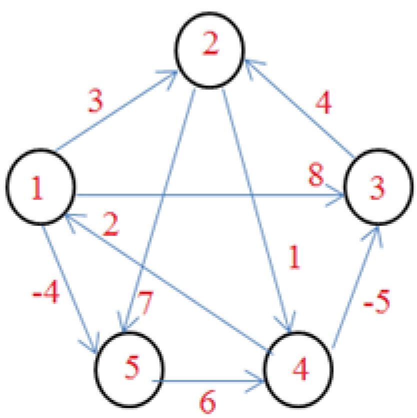
SECTION A

(5Qx 4M = 20 Marks)

S#	**Questions**	**Marks**	**COs**
Q1	For each of the following recurrences, give an expression for the runtime $T(n)$ if the recurrence can be solved with the Master Theorem. Otherwise, indicate that the Master Theorem does not apply. a. $T(n) = 2^n T(n/2) + n^n$ b. $T(n) = 16T(n/4) + n$ c. $T(n) = 2T(n/2) + n/\log n$ d. $T(n) = 2T(n/4) + n^{0.51}$	1+1+1+1 = 4	CO1
Q2	An array $A(n)$ contains n elements of the same value that means $A[1] = A[2] = A[3] = ... = A[n] = x$. Calculate the complexity of sorting $A(n)$ using quick sort?	4	CO2
Q3	a. Let G be a weighted connected undirected graph with distinct positive edge weights. If every edge weight is increased by the same value and the graph is updated as G', then what is the relation between graph G and G' in terms of their: i. Minimum Spanning Tree ii. Shortest path between any pair of vertices b. What will be the cost of the string if character c_i is at depth d_i and occurs at frequency f_i, where the number of distinct characters in the string are n?	1+1+2 = 4	CO3

Q4	What is the difference between linear sorting and comparison based sorting? Explain the different steps of the counting sort algorithm with an example. Discuss its time complexity.	1+2+1 = 4	CO5
Q5	Explain the P, NP, NP-Hard, and NP-complete classes and give the relation between them.	4	CO6
	SECTION B **(4Qx10M = 40 Marks)**		
Q6	a. Find the complexity of the following set loops, where n is given as input: $i \leftarrow n;$ $while\ (i\ >\ 1)\ \{$ $j\ =\ i; \quad //\%\%\ CAUTION:\ THIS\ DOES\ NOT\ START\ AT\ 0$ $while\ (j\ <\ n)\ \{$ $k \leftarrow 0;$ $while\ (k\ <\ n)\ \{$ $k \leftarrow k\ +\ 2;$ $\}$ $j \leftarrow j \times 2;$ $\}$ $i \leftarrow i\ /\ 2;$ $\}$ Express your answer using the $\theta(\cdot)$notation. b. An algorithm solves problems by dividing a problem of size n into 3 sub-problems of one-fourth the size and recursively solves the smaller sub-problems. It takes constant time to combine the solutions of the sub-problems. Calculate the runtime complexity of the solution by using either the iteration method or recurrence tree method.	5+5 = 10	CO1
Q7	We need to design a method for faster accessing of content in a private cloud which needs to sort the content of a file in the cloud itself. Since, space in the cloud needs to be utilized optimally, therefore we need to use a sorting algorithm which can do the task in-place without using any extra space. Suppose the indexes of the content in the file are stored in $A\ =\ [2,\ 8,\ 7,\ 1,\ 3,\ 5,\ 9]$. What would be the scenario after three iterations considering the optimal average case complexity. **OR**	10	CO2

	Suppose there is a message and the frequencies of the different alphabets are given as: b = 50, r = 10, a = 3, k = 30, e = 2, s = 5. What should be the representation of these different characters based on the Huffman algorithm?		
Q8	a. Being the winner of the annual technical fest of UPES, you get the chance of picking 5 distinct souvenirs from the desk, however your pockets allow you to carry only 60 units. The respective weight and value units associated with the souvenirs are <5, 10, 20, 30, 40> and <30, 20, 100, 90, 160>, respectively. Being in hurry and excitement, greedily you picked the items from the desk. Calculate the total value of the items that you would be able to pick from the desk. b. Briefly discuss the problem definition of “Longest Common Subsequence". Given two sequences X = ABCBDAB and Y = BDCABA, calculate the LCS of X and Y using dynamic programming.	5+5 = 10	CO3
Q9	a. Find all possible subsets of the sum to m. Let w= {3, 4, 5, 6} and m=9 and draw the state space tree that is generated. b. How many unique colors will be required for proper vertex coloring of: i. An empty graph having n vertices ii. A complete graph of n vertices iii. A cycle of n vertices iv. A bipartite graph having n vertices	5+5 = 10	CO5
SECTION C **(2Qx 20M= 40 Marks)**			
Q10	a. Consider the given graph and calculate the weight of the minimum spanning tree (MST) using the Prim’s algorithm? State and justify your explanation whether the MST is unique	10+10 = 20	CO3

	for the given graph.  b. Derive the recurrence relations of Best, Worst, and Average-case time complexities of the Quicksort algorithm.		
Q11	Consider the matrices P, Q, R and S which are 6 x 5, 5 x 7, 7 x 3 and 3 x 9, respectively. What is the minimum number of multiplications required to multiply the four matrices? Compute the optimal sequence and optimal parenthesization for matrix multiplication. Also design the algorithms for the optimal sequence and optimal parenthesization through analyzing the space and time complexity. **OR** In the given graph, what is the minimum cost to travel from vertex 1 to vertex 3 using the dynamic programming paradigm?  Also complete the program through analyzing the time complexity. $n = rows[W]$ $D(0) = W$ $for\ k = 1\ to\ n$ $do\ for\ i = 1\ to\ n$ $do\ for\ j = 1\ to\ n$ do ____________________ $return\ D(n)$	20	CO4