Name:
Enrolment No:

UPES
UNIVERSITY WITH A PURPOSE

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2021

Course: Statistical Modelling for Computer Sciences **Semester: I**
Program: M.Tech. (CSE) **Time : 03 hrs.**
Course Code: CSEG7003 **Max. Marks: 100**
No. of printed pages: 4

Instructions: Attempt all the questions. Refer appendix for required distribution tables.

Section A
(Scan and Upload) **(5Qx 4M = 20 Marks)**

S. No.		Marks	CO
Q1	Find the first four moments about the origin for a random variable X having density function $$f(x) = \begin{cases} 2x(9-x^3)/18 & 0 \le x \le 5 \\ 0 & otherwise \end{cases}$$	**[4]**	**CO3**
Q2	A continuous random variable X has probability density given by $$f(x) = \begin{cases} 5e^{-3x} & x > 0 \\ 0 & x \le 0 \end{cases}$$ Find (a) $E(X)$ (b) $E(X^2)$	**[4]**	**CO1**
Q3	X is random variable such that E(X) = 3 and E(X2) = 13. Determine a lower bound for P(- 2 < x < 8), using Chebyshev 's inequality?	**[4]**	**CO2**
Q4	How do Markov Chains work and what is memorylessness property?	**[4]**	**CO1**
Q5	Explain the basic queueing process? Discuss the Kendall notation of queue.	**[4]**	**CO5**

Section B
(Scan and Upload) **(4Qx10M = 40 Marks)**

Q6	Duracell manufactures batteries that the CEO claims will last an average of 300 hours under normal use. A researcher randomly selected 20 batteries from the production line and tested these batteries. The tested batteries had a mean life span of 270 hours with a standard deviation of 50 hours. Do we have enough evidence to suggest that the claim of an average lifetime of 300 hours is false? (Refer table as provided in Appendix)	**[10]**	**CO2**

<table>
<tr><td>Q7</td><td>A department store, A, has four competitors: B,C,D, and E. Store A hires a consultant to determine if the percentage of shoppers who prefer each of the five stores is the same. A survey of 1100 randomly selected shoppers is conducted, and the results about which one of the stores shoppers prefer are below. Is there enough evidence using a significance level $\alpha = 0.05$ to conclude that the proportions are really the same?
<table><tr><th>Store</th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th></tr><tr><td>Number of shoppers</td><td>262</td><td>234</td><td>204</td><td>190</td><td>210</td></tr></table>(Refer table as provided in Appendix)</td><td>[10]</td><td>CO3</td></tr>
<tr><td>Q8</td><td>Discuss the characteristics of Bernoulli trial. You are a telemarketer with a 10% chance of persuading a randomly selected person to switch to your long-distance company. You make 8 calls. What is the probability that exactly one is successful?
or
The average playing time of CDs in a large collection is 35 minutes, and the standard deviation is 5 minutes.
a) What value is 1 standard deviation above the mean? 1 standard deviation below the mean? What values are 2 standard deviations from the mean?
b) Assuming the distribution of time is approximately normal, about what percentage of times are between 25 and 45 minutes?</td><td>[10]</td><td>CO3</td></tr>
<tr><td>Q9</td><td>A hospital switch board receives an average of 4 emergency calls in a 10 min. interval. What is the probability that
(i) there are at the most 2 emergency calls and
(ii) there are exactly 3 emergency calls in a 10 min. interval?</td><td>[10]</td><td>CO2</td></tr>
<tr><td colspan="4">SECTION-C
(Scan and Upload) (2Qx 20M= 40 Marks)</td></tr>
<tr><td>Q10</td><td>A sports statistician claims that the mean winning times for Boston Marathon women’s open division champions is at least 2.68 hours. The mean winning time of a sample of 35 randomly selected Boston Marathon women’s open division champions is 2.60 hours. Assume the population standard deviation is 0.32 hour. (Refer table as provided in Appendix)
a) Identify the claim and state H_0 and H_a.
b) Find the standardized test statistic z.
c) Find the corresponding P value.
d) At alpha = 0.05, decide whether to reject or fail to reject the null hypothesis.
e) Interpret the decision in the context of the original claim</td><td>[20]</td><td>CO4</td></tr>
<tr><td>Q11</td><td>At Indian petrol pump, customers arrive according to a Poisson process with an average time of 5 minutes between arrivals. The service time is exponentially distributed with mean time = 2 minutes. On the basis of this information, find out
a) What would be the average queue length?
b) What would be the average number of customers in the queuing system?
c) What is the average time spent by a car in the petrol pump?
d) What is the average waiting time of a car before receiving petrol?

OR</td><td>[20]</td><td>CO5</td></tr>
</table>

	Punjab National Bank is considering opening a drive in window for customer service. Management estimates that customers will arrive at the rate of 15 per hour. The teller whom it is considering to staff the window can service customers at the rate of one every three minutes. Assuming Poisson arrivals and exponential service find a) Average number in the waiting line. b) Average number in the system. c) Average waiting time in line. d) Average waiting time in the system.		

Appendix

Standard Normal Cumulative Probability Table

Cumulative probabilities for NEGATIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379

TABLE D

t distribution critical values

	Upper-tail probability *p*											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073

Percentage Points of the Chi-Square Distribution

Degrees of Freedom	Probability of a larger value of x^2								
	0.99	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.01
1	0.000	0.004	0.016	0.102	0.455	1.32	2.71	3.84	6.63
2	0.020	0.103	0.211	0.575	1.386	2.77	4.61	5.99	9.21
3	0.115	0.352	0.584	1.212	2.366	4.11	6.25	7.81	11.34
4	0.297	0.711	1.064	1.923	3.357	5.39	7.78	9.49	13.28
5	0.554	1.145	1.610	2.675	4.351	6.63	9.24	11.07	15.09
6	0.872	1.635	2.204	3.455	5.348	7.84	10.64	12.59	16.81
7	1.239	2.167	2.833	4.255	6.346	9.04	12.02	14.07	18.48
8	1.647	2.733	3.490	5.071	7.344	10.22	13.36	15.51	20.09
9	2.088	3.325	4.168	5.899	8.343	11.39	14.68	16.92	21.67
10	2.558	3.940	4.865	6.737	9.342	12.55	15.99	18.31	23.21
11	3.053	4.575	5.578	7.584	10.341	13.70	17.28	19.68	24.72
12	3.571	5.226	6.304	8.438	11.340	14.85	18.55	21.03	26.22
13	4.107	5.892	7.042	9.299	12.340	15.98	19.81	22.36	27.69
14	4.660	6.571	7.790	10.165	13.339	17.12	21.06	23.68	29.14
15	5.229	7.261	8.547	11.037	14.339	18.25	22.31	25.00	30.58
16	5.812	7.962	9.312	11.912	15.338	19.37	23.54	26.30	32.00
17	6.408	8.672	10.085	12.792	16.338	20.49	24.77	27.59	33.41
18	7.015	9.390	10.865	13.675	17.338	21.60	25.99	28.87	34.80
19	7.633	10.117	11.651	14.562	18.338	22.72	27.20	30.14	36.19
20	8.260	10.851	12.443	15.452	19.337	23.83	28.41	31.41	37.57