Name:

Enrolment No:

UPES
UNIVERSITY WITH A PURPOSE

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2021

Course: **Probability Theory & Statistics** **Semester: V**
Program: **B.Sc. (Hons.) Mathematics** **Time: 03 hrs.**
Course Code: MATH 3013 **Max. Marks: 100**
Instructions: All questions are compulsory.

SECTION A (Each question carries 4 marks)

S. No.		**Marks**
Q1	Let the first four moments of a distribution about the value 5 be 2, 20, 40 and 50 then find the Variance of the distribution.	**CO1**
Q2	If X represents the outcome, when a fair die is tossed, then evaluate the moment generating function of X.	**CO1**
Q3	A random variable X has an exponential distribution with probability density function given by $f(x) = 3e^{-3x}, for\ x > 0$ and zero elsewhere then determine the probability that X is not less than 5.	**CO2**
Q4	If $f\ (x, y) = k\ (1 - x)(1 - y), 0 < x, y < 1$, is a joint density function then find the value of k.	**CO3**
Q5	The transition probability matrix of a Markov chain $\{X_n\}, n = 1, 2, 3$ .... Having three states 1, 2 and 3 is $p = \begin{matrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{matrix}$ and the initial distribution is $p^{(0)} = (0.7, 0.2, 0.1)$ then evaluate $P\ \{X_1 = 3, X_0 = 2\}$.	**CO5**

SECTION B (Each question carries 10 marks)

S. No.		**Marks**
Q6	If 10% of the bolts produced by a machine are defective, determine the probability that out of 10 bolts chosen at random (a) One bolt will be defective. (b) None will be defective. (c) At most two bolts will be defective.	**CO2**
Q7	Three balls are drawn at random without replacement from a box containing 2 white, 3 red and 4 black balls. If X denotes the number of white balls drawn and Y denotes the number of red balls drawn, find the joint probability distribution of (X, Y).	**CO3**
Q8	Examine if the weak law of large numbers holds for the sequence $\{X_p\}$ of independent identically distributed random variables with $P[X_k = (-1)^{k-1}.k] = \frac{6}{\pi^2 k^2}, k = 1, 2, \dots; p = 1, 2, \dots..$	**CO4**

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<td>Q9</td>
<td>A fair dice is 720 times. Use Chebyshev's inequality to find a lower bound for the probability of getting 90 to 150 sixes.
OR
If $X_1, X_2, X_3, \dots \dots \dots \dots X_n$ are Poisson variate with parameter lambda is equal to 3, Use the central limit theorem to estimate $P\ (220 \leq S_n \leq 260)$, where $S_n = X_1 + X_2 + X_3 \dots \dots \dots \dots + X_n\ and\ n = 75$.</td>
<td>CO4</td>
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<tr>
<td colspan="3">SECTION-C (Each question carries 20 marks)</td>
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<td>Q 10</td>
<td>Define Central limit theorem and if lifetime of a certain brand of an electric bulb may be considered a random variable with mean 1000 hours and standard deviation 200 hours. Find the probability, using the same that the average lifetime of 50 bulbs exceeds 1300 hours.</td>
<td>CO4</td>
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<td>Q 11</td>
<td>Calculate the coefficient of correlation and obtain the lines of regression for the following data:
<table>
<tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr>
<tr><td>y</td><td>9</td><td>8</td><td>19</td><td>12</td><td>11</td><td>13</td><td>14</td><td>16</td><td>15</td></tr>
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OR
The joint probability mass function of (X, Y) is given by $P\ (x, y) = k\ (2x + 3y),\ x = 0, 1, 2;\ y = 1, 2, 3$. Find all the marginal and conditional probability distributions. Also find the probability distribution of (X + Y).</td>
<td>CO3</td>
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