Name:

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UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2021

Course: Numerical Methods **Semester: V**
Program: B.Sc. (Hons.) Mathematics **Time : 03 hrs.**
Course Code: MATH 3021 **Max. Marks: 100**
Instructions: Scientific calculator is allowed.

SECTION A
(5 X 4 = 20)
All questions are compulsory.

S. No.		Marks	CO
Q1	Evaluate the smallest positive root of $x^3 - 5x + 3 = 0$ by Newton Raphson Method.	4	CO1
Q2	Find the missing terms in the given table X: 45, 50, 55, 60, 65 Y: 3.0, ---, 2.0, ---, -2.4	4	CO2
Q3	Evaluate $\Delta^{10}[(1-ax)(1-bx^2)(1-cx^3)(1-dx^4)$	4	CO2
Q4	Using Trapezoidal rule, calculator $\int_0^1 \frac{1}{1+x^2} dx$ using 5 intervals.	4	CO3
Q5	Use Euler's method to solve for $y(0.4)$, considering step-length $h = 0.2$, given that $\frac{dy}{dx} = 1 + y^2$, with initial condition $y(0) = 0$.	4	CO5

SECTION B
(4 X 10 = 40)
Q 1-Q3 are compulsory and Q4 have internal choice

Q1	Differentiate between round off error and truncation error with help of an example. The diameter and altitude of a can in the shape of a right circular cylinder are measured 4cm and 6cm respectively. The possible error in each measurement is 0.01cm. Find the maximum possible relative error in computation of its volume.	10	CO1
Q2	Three neighbors make a fruit salad for a summer picnic. One person uses three pounds of strawberries plus five pounds of grapes plus one pound of melon at a cost of \$20. A second person uses three pounds of strawberries plus two pounds of grapes plus two pounds of melon at a cost of \$21. The last neighbor uses four pounds of strawberries plus three pounds of grapes plus three pounds of melon at a cost of \$30. Using Gauss Jordan method find how much does each fruit cost?	10	CO4
Q3	Solve the following using Gauss Seidal Method. $-2x_1 + 3x_2 + 10x_3 = 22$ $2x_1 + 20x_2 - 2x_3 = -44$ $10x_1 + 2x_2 + x_3 = 9$	10	CO4

Q4

Use Lagrange's Interpolation formula to fit a polynomial to the data. Hence or otherwise find the value of U_1

X	-1	0	2	3
U_x	-8	3	1	12

OR

Using Newton Divided Difference formula find the polynomial $f(x)$ with the help of following table and evaluate $f(1)$

x	-4	-1	0	2	5
$f(x)$	1245	33	5	9	1335

10 | CO2

SECTION-C
(2 X 20 = 40)
Q1 is compulsory and Q2 have internal choices

Q1

(a) Use fourth order Runge-Kutta method to solve for y(1.2), considering step-length h = 0.1, given that

$$\frac{dy}{dx} = x^2 + y^2, \qquad y(1) = 1.5.$$

(b) Use Modified Euler's method to solve for y(0.4), considering step-length h = 0.2, given that

$$\frac{dy}{dx} = 1 + y^2, \;\; y(0) = 0.$$

10+10 | CO5

Q2

(a) Evaluate $\int_1^2 \int_2^3 e^{x+y} dxdy$ using composite 1/3 Simpson Rule and compare with exact solution.

OR

A solid of revolution is formed by rotating about the x-axis the area between the x-axis, the lines x = 0 and x =1, and a curve through the points with the following coordinates

x	0.00	0.25	0.50	0.75	1.00
y	1.0000	0.9896	0.9589	0.9089	0.8415

Estimate the volume of the solid formed using Simpson's rule.

(b) A slider in a machine moves along a fixed straight rod. Its distance x cm along the rod is given below for various values of the time t. Find the velocity and acceleration of the slider when i): = 0.1 second ii) t = 0.6 second.

t	0	0.1	0.2	0.3	0.4	0.5	0.6
x	30.13	31.62	32.87	33.64	33.95	33.81	33.24

OR

From the following table, find x, correct to two decimal places for which y is max and find this value of y

x	1.2	1.3	1.4	1.5	1.6
y	0.9320	0.9636	0.9855	0.9975	0.9996

10+10 | CO3