| Name: <br> Enrolment No: |  |  |  |
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| Cours <br> Progr <br> Cours | UNIVERSITY OF PETROLEUM AND ENERGY STUDIES End Semester Examination, December 2021 <br> Mathematical Physics <br> : B.Sc. (H) Physics and Integrate B.Sc. M.Sc. Physics <br> Code: PHYS 1011 |  | $\begin{aligned} & \text { ter: I } \\ & \mathbf{3} \text { hrs. } \\ & 100 \end{aligned}$ |
| 1. Each Question will carry 4 Marks SECTION A <br> 2. Instruction: Write the statement / Select the correct answer(s) <br> S. |  |  |  |
| S. No. |  | Marks | CO |
| Q1 | What are the key differences between Normal, Binomial, and Poisson Distribution? | 4 | CO1 |
| Q2 | A population grows at a rate of $5 \%$ per year. How long does it take for the population to double? | 4 | CO 2 |
| Q3 | Solve the differential equation $\left(x^{2}+y^{2}+2 x\right) d x+2 y d y=0$. | 4 | CO 2 |
| Q4 | The curl of vector field $\vec{f}(x, y, z)=x^{2} \hat{i}+2 z \hat{j}-y \hat{k}$ is ? | 4 | CO3 |
| Q5 | Find the particular integral of $\left(D^{3}-3 D^{2}+4\right) y=e^{2 x}$. | 4 | CO2 |
| SECTION B <br> 1. Each question will carry 10 marks <br> 2. Instruction: Write short / brief notes |  |  |  |
| Q6 | a) If A is a Hermitian (skew-Hermitian) matrix, show that iA is a skew Hermitian (Hermitian) matrix. <br> b) Define Dirac Delta function and list its important properties. | 10 | CO1 |
| Q7 | a) Define differentiability of a function at a given point. What is the necessary and sufficient condition for a function to be differentiable (State mathematically)? <br> b) Write the expressions for the 'del' operator in Cartesian, cylindrical and spherical coordinates. | 10 | CO1 |
| Q8 | If $\vec{r}=x \hat{\imath}+y \hat{\jmath}+z \hat{k}$, then show that <br> a) $\vec{\nabla} r=\frac{\vec{r}}{r}$ <br> b) $\vec{\nabla}\left(\frac{1}{r}\right)=-\frac{\vec{r}}{r^{3}}$ | 10 | CO3 |
| Q9 | Find complete solution of any of the following differential equations: | 10 | CO 2 |


|  | $(1+x)^{2} \frac{d^{2} y}{d x^{2}}+(1+x) \frac{d y}{d x}+y=\sin [2 \log (1+x)]$ |  |
| :---: | :---: | :---: |
| OR |  |  |
|  | $\left(D^{2}-4 D+4\right) y=8 x^{2} e^{2 x} \sin 2 x$ |  |

## SECTION-C

## 1. Each Question carries 20 Marks.

2. Instruction: Write long answers.

| Q10 | (a) Find the angle between the surfaces $z=x^{2}+y^{2}$ and $z=\left(x-\frac{\sqrt{6}}{6}\right)^{2}+\left(y-\frac{\sqrt{6}}{6}\right)^{2}$ at the point $P=\left(\frac{\sqrt{6}}{12}, \frac{\sqrt{6}}{12}, \frac{1}{12}\right)$. <br> (b) Find the constants $a, b, c$ so that $\vec{F}=(x+2 y+a z) \hat{\imath}+(b x-3 y-z) \hat{j}+(4 x+c y+2 z) \hat{k}$ is irrotational and hence find the function $\varphi$ such that $\overrightarrow{F \vec{\nabla}} \varphi$ | 20 | CO3 |
| :---: | :---: | :---: | :---: |
| 11 | (a) Calculate the work done in moving a particle once around a circle C in the xy plane if the circle has center at the origin and radius 3 and if the force field is given by $\vec{F}=(2 x-y+z) \hat{\imath}+\left(x+y-z^{2}\right) \hat{j}+(3 x-2 y+4 z) k$. Does the work done in this case depend on path? Justify your answer. <br> (b) Evaluate $\iint_{S} x y^{2} d x d y$ <br> on the surface given below: | 20 | CO4 |


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| [Either do above (a) and (b) both, or the following] |  |
| Suppose $\mathbf{F}=y \mathbf{i}+(x-2 x z) \mathbf{j}-x y \mathbf{k}$. Evaluate $\iint_{\mathbf{R}}(\boldsymbol{\nabla} \times \mathbf{F}) \cdot \mathbf{n} d S$ where $S$ is the surface of the sphere |  |
| $x^{2}+y^{2}+z^{2}=a^{2}$ above the $x y$-plane |  |

