UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, January 2022

Course: Algebra | **Semester : I**

Program: B.Sc. (Hons.) Mathematics & Int. B.Sc. & M.Sc. (Mathematics) | **Duration : 03 hrs.**

Course Code: MATH 1040 | **Max. Marks: 100**

Instructions:

1. Section A has 5 questions. All questions are compulsory.
2. Section B has 4 questions. All questions are compulsory. Question 4 has internal choice to attempt any one.
3. Section C has 2 questions. All questions are compulsory. Question 2 has internal choice to attempt any one.

SECTION A

(Scan and upload) **(5Qx 4M = 20 Marks)**

		Marks	COs
Q 1	For what values of k the complex number $Z_1 = 2e^{\frac{\pi}{3}i}$ and $Z_2 = 2e^{\frac{6k\pi+\pi}{3}i}$ are equal?	**4**	**CO1**
Q 2	Find the modulus, argument, and polar form of the complex number $$Z = -3i.$$	**4**	**CO1**
Q 3	Using mathematical induction, show that if n is a positive integer then $$1 + 2 + \cdots + n = \frac{n(n+1)}{2}.$$	**4**	**CO2**
Q 4	Find the rank of the matrix $$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \\ 16 & 17 & 18 & 19 & 20 \\ 21 & 22 & 23 & 24 & 25 \end{bmatrix}.$$	**4**	**CO3**
Q 5	Calculate the values of k such that the system of equations $$x + ky + 3z = 0, \quad 4x + 3y + kz = 0, \quad 2x + y + 2z = 0$$ has non-trivial solution.	**4**	**CO3**

SECTION B

(Scan and upload) **(4Qx10M = 40 Marks)**

		Marks	COs
Q 1	Determine all the roots of $\left(-8 - 8\sqrt{3}\, i\right)^{1/4}$ and exhibit them geometrically.	**10**	**CO1**
Q 2	The linear transformation $F: \mathbb{R}^2 \to \mathbb{R}^2$ is defined by $$F(x, y) = (2x + 3y, 4x - 5y),$$ where $\mathbb{R}$ is the set of real number. Find the matrix representation $[F]_S$ of F relative to the basis $S = \{u_1, u_2\} = \{(1, -2), (2, -5)\}$.	**10**	**CO4**
Q 3	Let the matrix A be given as $$A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 3 & 1 & 2 & 1 \\ 4 & 6 & 2 & -4 \\ -6 & 0 & -3 & -4 \end{bmatrix}.$$ Check whether the rows of matrix A form a set of independent vectors. If not then find the relation among them.	**10**	**CO3**

<table>
<tr><td>Q 4</td><td>Define the eigenvalues and eigenvectors of a square matrix. Find the eigenvalues and eigenvectors of the matrix A, which is given as
$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}.$$
OR
Find for what values of λ and μ the system of linear equations:
$$x + y + z = 6$$ $$x + 2y + 5z = 10$$ $$2x + 3y + \lambda z = \mu$$
has (i) a unique solution, (ii) no solution, (iii) infinite solutions. Also find the solution for $\lambda = 2$ and $\mu = 8$.</td><td>10</td><td>CO3</td></tr>
<tr><td colspan="4">SECTION-C
(Scan and upload) (2Qx 20M= 40 Marks)</td></tr>
<tr><td>Q 1</td><td>(a) State and prove division algorithm.
(b) Use Euclidean algorithm to find greatest common divisor of integers 242 and 758.</td><td>20</td><td>CO2</td></tr>
<tr><td>Q 2</td><td>Define vector space.
Show that the set $\mathbb{R}^n = \{(a_1, a_2, \dots, a_n) \mid a_1, a_2, \dots, a_n \in \mathbb{R}\}$ is vector space over the field $\mathbb{R}$, where $\mathbb{R}$ is the set of real numbers.
OR
Give the definition of linear transformation.
Let $F: \mathbb{R}^4 \to \mathbb{R}^3$ be the linear transformation defined by
$F(x, y, z, t) = (x - y + z + t, 2x - 2y + 3z + 4t, 3x - 3y + 4z + 5t)$
where $\mathbb{R}^4$ and $\mathbb{R}^3$ are the vector space over the set of real number $\mathbb{R}$.
(a) Find a basis and the dimension of the image of F.
(b) Find a basis and the dimension of the kernel of F.</td><td>20</td><td>CO4</td></tr>
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