UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, January 2022

Course: Finite Element Methods for Fluid Dynamics **Semester: I**

Program: M. Tech CFD **Time: 03 hrs.**

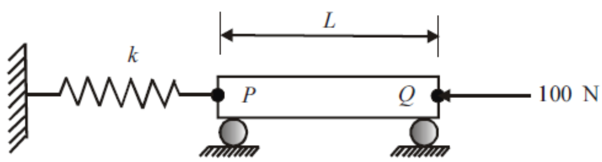
Course Code: ASEG 7022 **Max. Marks: 100**

Pages: 04

Instructions: Make use of sketch/plots to elaborate your answer. All sections are compulsory

SECTION A

(Scan and upload) **(5Qx 4M = 20 Marks)**

S. No.		Marks	CO
Q 1	Consider the spring-mounted bar as shown in the figure. Solve for the displacements of points P and Q using the bar elements (assume AE = constant).  	**[04]**	**CO2**
Q 2	Define the following terms: (i) Shell element; (ii) Beam element; (iii) Truss element; (iv) 3D element	**[04]**	**CO1**
Q 3	Identify the natural and essential boundary conditions of the following differential equation: $\frac{d^2}{dx^2}\left[a(x)\frac{d^2y}{dx^2}\right]+b(x)=0$ for $0<x<L$; subject to the following boundary conditions: $y=0$ and $dy/dx=0$ at $x=0$; $\left[a(x)\frac{d^2y}{dx^2}\right]_{x=L}=A$, $\left[\frac{d}{dx}\left(a(x)\frac{d^2y}{dx^2}\right)\right]_{x=L}=0$	**[04]**	**CO1**
Q 4	Consider the following heat conduction problem $(0 \le x \le 1)$: $\frac{d}{dx}\left(k\frac{dT}{dx}\right)+S=0$ The boundary conditions specified are as follows: $T(1)=\sqrt{2}$, $\left(\frac{dT}{dx}\right)_{x=0}=0$ (i) Is $T = Cos(\pi x) + Sec(\frac{\pi x}{4})$ a valid trial function? Explain. (ii) Is $T = Sin(\pi x) + Cosec(\pi x)$ a valid weighting function? Explain.	**[04]**	**CO2**

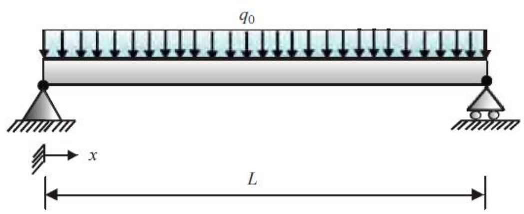
Q 5	Explain why arbitrarily oriented mechanical loads on an idealized pin-jointed truss structure must be applied at the joints. [Hint: idealized truss members have no bending resistance.] How about actual trusses: can they take loads applied between joints?	**[04]**	**CO3**

SECTION B

(Scan and upload) **(4Qx10M = 40 Marks)**

Q 6 Consider a simply supported beam under uniformly distributed load as shown in figure. The governing differential equation and the boundary conditions are given by

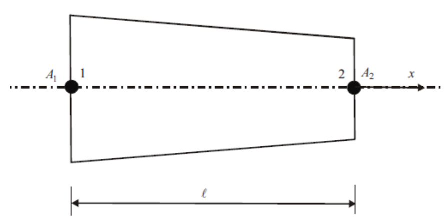
$$EI\frac{d^4v}{dx^4} - q_0 = 0 \qquad v(0) = 0, \frac{d^2v}{dx^2}(0) = 0, v(L) = 0, \frac{d^2v}{dx^2}(L) = 0$$



Find the approximate solution using the point collocation technique at $x = L/2$.

[10] **CO2**

Q 7 Consider a bar element whose area of cross-section varies linearly along the longitudinal axis. Derive its stiffness matrix. How will this compare with the stiffness matrix obtained assuming that the bar is of uniform cross section area equal to that of its mid-length?



[10] **CO3**

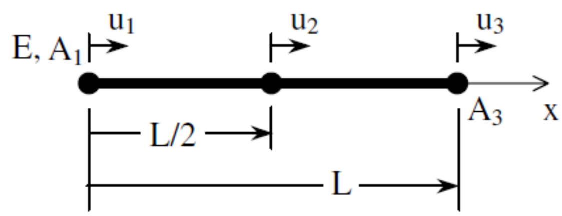
Q 8 A 3 node rod element has a quadratic shape function matrix:

$$N = \langle 1 - \frac{3x}{L} + \frac{2x^2}{L^2}, \frac{4x}{L} - \frac{4x^2}{L^2}, -\frac{x}{L} + \frac{2x^2}{L^2} \rangle$$

For $L = 1\ m$, $E = 200 \times 10^9$ Pa, $u_1 = 0, u_2 = 5 \times 10^{-6} m, u_2 = 15 \times 10^{-6} m$

Find:

a. The displacement u at $x = 0.25\ m$.
b. The strain as a function of x.
c. The strain at $x = 0.25\ m$.
d. The stress at $x = 0.25\ m$



[10] **CO3**

Q 9 [10] CO4

Considering the following force displacement relationship,

$$\mathbf{f} = \mathbf{K}\mathbf{u} = \begin{bmatrix} 20 & 10 & -10 & 0 & -10 & -10 \\ 10 & 10 & 0 & 0 & -10 & -10 \\ -10 & 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & -5 \\ -10 & -10 & 0 & 0 & 10 & 10 \\ -10 & -10 & 0 & -5 & 10 & 15 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.4 \\ -0.2 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \\ 0 \\ 1 \\ 2 \\ 1 \end{bmatrix}$$

Draw a free body diagram of the nodal forces acting on the free-free truss structure and verify that this force system satisfies translational and rotational (moment) equilibrium.

OR

Solve the differential equation for a physical problem expressed as $\frac{d^2y}{dx^2} + 100 = 0$

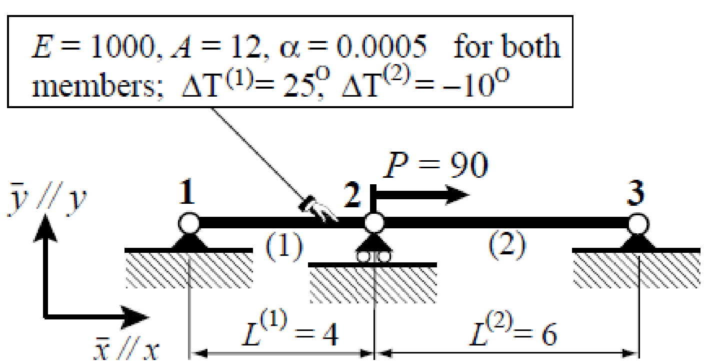
$0 \leq x \leq 10$ with boundary conditions as y(0)=0 and y(10)=0 using

(i) Point collocation method
(ii) Sub domain collocation method

SECTION-C
(Scan and upload) **(2Qx 20M= 40 Marks)**

Q 10 [20] CO5

Two truss members are connected in series as shown in fig and fixed at the ends. Properties $E = 1000$, $A = 12$ and $\alpha = 0.0005$ are common to both members. The member lengths are 4 and 6. A mechanical load $P = 90$ acts on the roller node. The temperature of member (1) increases by $\Delta T(1) = 25^0$ while that of member (2) drops by $\Delta T(2) = -10^0$. Find the stress in both members.



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<td>Q 11</td>
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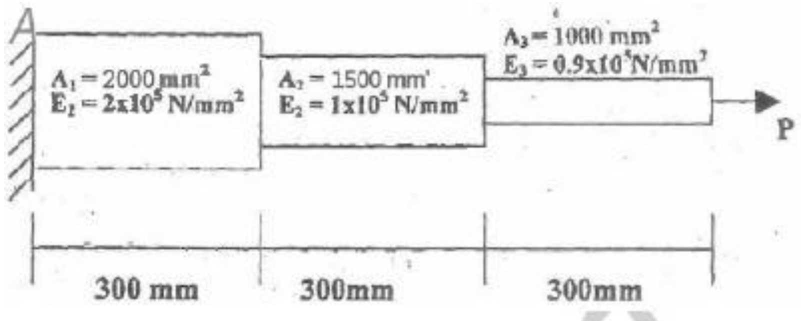
Consider the bar shown in figure axial force P = 30KN is applied as shown. Determine the nodal displacement, stresses in each element and reaction forces.



OR

Derive the equivalent spring formula *F = (EA/L) d* by the Theory of Elasticity relations $e = d\bar{u}(\bar{x})/d\bar{x}$ (strain-displacement equation), $\sigma = Ee$ (Hooke's law) and $F = A\sigma$ (axial force definition). Here *e* is the axial strain (independent of *x*) and σ the axial stress (also independent of *x*). Finally, *u(x)* denotes the axial displacement of the cross section at a distance *x* from node *i* , which is linearly interpolated as

$$\bar{u}(\bar{x}) = \bar{u}_{xi}\left(1 - \frac{\bar{x}}{L}\right) + \bar{u}_{xj}\frac{\bar{x}}{L}$$

Justify above equation is correct since the bar differential equilibrium equation: $d[A(d\sigma/d\bar{x})]/d\bar{x} = 0$, is verified for all *x* if *A* is constant along the bar.
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<td>[20]</td>
<td>CO5</td>
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