Name: **Enrolment No:**	UPES UNIVERSITY WITH A PURPOSE

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examinations, December 2021 - January 2022

Course: Remedial Mathematics **Semester: I**
Program: B.Sc/B.Tech/Int.B.Sc+M.Sc/Int.B.Tech+MBA **Time : 03 hrs.**
Course Code: BP106RMT **Max. Marks: 100**

SECTION A (Answer all the questions)

S. No.	Answer all the questions. Each carries 1.5 marks.	**30 Marks**	**CO**
1	Find the determinant of the matrix $A = \begin{bmatrix} 1 & 6 & 7 \\ 2 & 3 & 0 \\ 0 & 1 & 4 \end{bmatrix}$.	**1.5**	**CO1**
2	If the matrix $A = \begin{bmatrix} 1 & 4 \\ x & 8 \end{bmatrix}$ is singular, find the value of x.	**1.5**	**CO1**
3	If $A = [1\ 3]$ and $B = \begin{bmatrix} 3 & -2 & 1 & -3 \\ -1 & 4 & -3 & 2 \end{bmatrix}$ find AB.	**1.5**	**CO1**
4	If $A = \begin{bmatrix} 5 & 3 \\ -2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -3 \\ 4 & 8 \end{bmatrix}$, find $adj(A).adj(B)$.	**1.5**	**CO1**
5	If $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 4A + 3I = 0$.	**1.5**	**CO1**
6	Find the Eigen values of the matrix $A = \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}$.	**1.5**	**CO1**
7	Define proper and improper fractions with relevant examples.	**1.5**	**CO1**
8	If $\frac{3x+1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$, find the value of A.	**1.5**	**CO1**
9	Find the cofactors of the elements 6, 2 and 8 in $\begin{vmatrix} 3 & -4 & 6 \\ 2 & 5 & 7 \\ 9 & 8 & 0 \end{vmatrix}$.	**1.5**	**CO1**
10	Define even and odd functions with relevant examples.	**1.5**	**CO2**
11	Evaluate $\lim_{x \to 5} \frac{x^2-25}{x-5}$.	**1.5**	**CO2**
12	If $f(x) = \begin{cases} x+1 \ for \ x \leq 0 \\ x+2 \ for \ x > 0 \end{cases}$, check whether the $\lim_{x \to 0} f(x)$ exists or not.	**1.5**	**CO3**
13	Find value of $lim_{x \to 2} \left[\frac{x^3-2x^2}{x^2-4}\right]$.	**1.5**	**CO3**
14	Define (a) Differential Equation (b) Order and degree of Differential Equation with examples.	**1.5**	**CO3**
15	Find the general solution of the differential equation $\frac{dy}{dx} = xy$.	**1.5**	**CO3**

16	Find the derivative of $y = ax + b$ using the first principles of differentiation.	**1.5**	**CO3**
17	Find the value of $f'(3)$ if $f(x) = \frac{2x+3}{5x+2}$.	**1.5**	**CO3**
18	If $u = e^x$ and $v = \sin 2x$, find $\frac{d}{dx}(u.v)$.	**1.5**	**CO2**
19	If $y = (x^2 + 5x + 6)^8$, find the value of $\frac{dy}{dx}$.	**1.5**	**CO3**
20	Evaluate $\int \cos(ax + b)\ dx$ using substitution technique.	**1.5**	**CO3**
	SECTION B (Answer all the questions)		
Q	Short Answer Type Question (5 marks each) Scan and Upload 4 questions 5 marks each	**20 Marks**	**CO**
1	Find the adjoint of the matrix $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 2 & 1 \\ -4 & 5 & 2 \end{bmatrix}$.	**5**	**CO1**
2	Show that the equation $(3x^2y^2 + x^2)dx + (2x^3y + y^2)dy = 0$ is exact and hence solve it.	**5**	**CO4**
3	Evaluate the following limits $\lim_{x\to -1} \frac{x+1}{\sqrt{x+5}-2}$ (b) $\lim_{x\to 4} \frac{x^2-x-12}{x^2+x-20}$	**5**	**CO3**
4	Evaluate $\int \frac{2x+3}{(x-1)(x-2)}\ dx$ using the technique of partial fractions.	**5**	**CO2**
	SECTION C (Answer all the questions)		
Q	**Two case studies 15 marks each.**	**30 Marks**	**CO**
1	Case Study 1: **(Matrices)** (a) State Cayley-Hamilton Theorem and verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. [5 marks] (b) Solve the system of equations using Cramer's rule. $2x + 8y + 5z = 5; x + y - z = -2; x + 2y - z = 2$ [10 marks]	**15**	**CO2**

2	Case Study 2: **(Differentiation and Integration)** (a) Prove that $\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2+b^2}[a \sin bx - b \cos bx] + C$. [10 marks] (b) Define Chain rule. If $y = 3u^2 - 6$ and $u = 5x^2 + 2x$, find $\frac{dy}{dx}$ at $x = 5$. [5 marks]	**15**	**CO3**
	SECTION- D (Answer all the questions)		
Q	Long Answer type Questions Scan and Upload (10 marks each)	**20 Marks**	**CO**
1	Solve $y'' - 2y' - 8y = 0, y(0) = 3, y'(0) = 6$ using Laplace Transform.	**10**	**CO4**
2	Find (a) stationary points (b) maxima and minima for the function $f(x) = 3x^3 - 9x^2 - 27x + 15$.	**10**	**CO4**