

Name:

Enrollment No:



## UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, May 2021

Programme Name: B.Tech APE-UP

Course Name : Statistical Methods in Petroleum Engineering

Course Code: MATH 3029

Semester : VI

Time : 03 hrs

Max. Marks : 100

| <b>Section A</b><br>(All questions are compulsory. Tables are given at the end of the paper)                       |   |      |     |
|--|---|------|-----|
| 1.   | A bowler bowls on average 2 wickets per over following a Poisson process. Let $X$ be the waiting time in terms of number of overs until the first five wickets are bowled. Write the mean and variance of $X$ .   | [5]  | CO1 |
| 2.   | Let $X$ be a random variable following an exponential distribution with $\lambda = 1.4$ minutes. Let $X$ measures the time between cars passing a particular junction. You wait for three minutes, and start observing the cars passing the junction again. Write the probability that a car will pass in the next half minute? | [5]  | CO2 |
| 3.   | Consider a sample 5, 8.5, 12, 15, 7, 9, 7.5, 6.5, 10.5 from a population following normal distribution with unknown mean and variance 4. Write a 95 percent confidence interval for the mean.   | [5]  | CO3 |
| 4.   | Error in certain measurement is normally distributed with unknown mean and standard deviation 2. You believe that the average measurement should be 8. To test this belief five independent measurements are done and average value was found as 9.5. Is your hypothesis accepted at 5 percent level of significance?           | [5]  | CO4 |
| 5.   | Assuming second order stationary condition and intrinsic hypothesis, write relation between semivariogram and covariance functions.   | [5]  | CO5 |
| 6.   | In which kriging $E[Z(x)]$ is assumed constant and known.   | [5]  | CO5 |
| <b>SECTION B</b><br>(Q1-Q5 are compulsory and Q5 has an internal choice. Tables are given at the end of the paper) |   |      |     |
| 1.   | Consider sequences of coin flips. Each flip in a sequence is independent of other flips in the sequence. Head and tail are equally likely in each flip. Let $X$ be a random variable denoting the number of flips before a head appear for the first time. Find the probability mass function of the random variable $X - 1$ .  | [10] | CO1 |

Name:

Enrollment No:



|    |  |      |     |    |   |    |    |    |    |    |    |    |   |    |    |   |    |   |   |    |    |    |    |      |     |
|----|--|------|-----|----|---|----|----|----|----|----|----|----|---|----|----|---|----|---|---|----|----|----|----|------|-----|
| 2. | Let us consider time series of two climate variables, say it, temperature and precipitation. Explain with an example, how you will form joint distribution of these two random variables. Write two advantages of the joint distribution citing the temperature and precipitation.   | [10] | CO2 |    |   |    |    |    |    |    |    |    |   |    |    |   |    |   |   |    |    |    |    |      |     |
| 3. | Use the method of least squares to fit a regression line to the accompanying data points.<br><table border="1" data-bbox="305 520 1299 598"><tbody><tr><td>x</td><td>-1</td><td>0</td><td>2</td><td>-2</td><td>5</td><td>6</td><td>8</td><td>11</td><td>12</td><td>-3</td></tr><tr><td>y</td><td>-5</td><td>-4</td><td>2</td><td>-7</td><td>6</td><td>9</td><td>13</td><td>21</td><td>20</td><td>-a</td></tr></tbody></table> Where $a=d/10$ , where $d$ is the three digit number formed by last three digits of your roll number. For example if your roll number is R870218125, then $a = 12.5$ . Test if the slope is significant at 0.05.   | x    | -1  | 0  | 2 | -2 | 5  | 6  | 8  | 11 | 12 | -3 | y | -5 | -4 | 2 | -7 | 6 | 9 | 13 | 21 | 20 | -a | [10] | CO3 |
| x  | -1   | 0    | 2   | -2 | 5 | 6  | 8  | 11 | 12 | -3 |    |    |   |    |    |   |    |   |   |    |    |    |    |      |     |
| y  | -5   | -4   | 2   | -7 | 6 | 9  | 13 | 21 | 20 | -a |    |    |   |    |    |   |    |   |   |    |    |    |    |      |     |
| 4. | In mild rainy conditions optical depth of two different types of clouds (cloud A and cloud B) are measured as following:<br>Cloud A: 36, 42, 40, 51, 37, 36, 35, 40, 53, 48, 50, 52, 55, $a$<br>Cloud B: 58, 61, 63, 64, 59, 60, 65, 63, 68, 70, 78, 65, 59, $\frac{3a}{2}$<br>where $a$ is the three digit number formed by last three digits of your roll number. For example if your roll number is R870218125, then $a = 125$ .<br>Suppose that it is known that the optical depth of Cloud A in mild rainy conditions is normally distributed with unknown mean $\mu_1$ and known variance $\sigma_1^2 = 40$ , whereas the corresponding distribution for Cloud B is normal with unknown mean $\mu_2$ and known variance $\sigma_2^2 = 100$ . Determine a 95 percent confidence interval for $\mu_1 - \mu_2$ .  | [10] | CO4 |    |   |    |    |    |    |    |    |    |   |    |    |   |    |   |   |    |    |    |    |      |     |
| 5. | A geologist claims that mean temperature in certain region inside the Earth in kelvin is 345K. To verify the claim, following temperatures are obtained at randomly selected locations in the region: 340, 356, 332, 362, 318, 344, 386, 402, 322, 360, 362, 354, 340, 372, 338, 375, 364, 355, 324, and 370. Do the data contradict the geologist's claim?<br><br><b>OR</b><br><br>(a) Suppose the variance $s^2$ of 18 random measurements from a normal population is 0.68. The measurements are close to correct values if the variance is not greater than 0.36. Test the null hypothesis if the measurements are close to the correct values at 0.05 level of significance.<br>(b) In comparing the variability of two measurements, following is the summary of the collected data. $n_1 = 13$ , $s_1^2 = 19.2$ , $n_2 = 16$ , and $s_2^2 = 3.5$ . Measurements are independent random samples from two populations following normal distributions. Test the hypothesis for equality of the variances of the two populations at 0.02 level of significance. | [10] | CO4 |    |   |    |    |    |    |    |    |    |   |    |    |   |    |   |   |    |    |    |    |      |     |

Name:

Enrollment No:



**SECTION C**

**(Q1 is compulsory and has an internal choice. Tables are given at the end of the paper)**

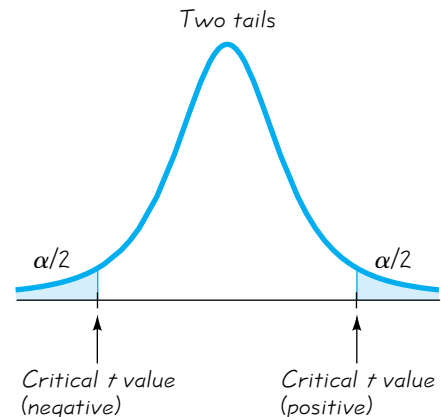
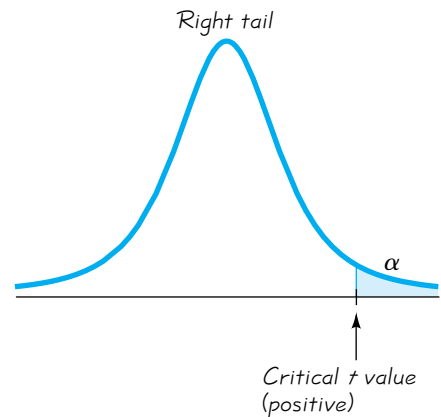
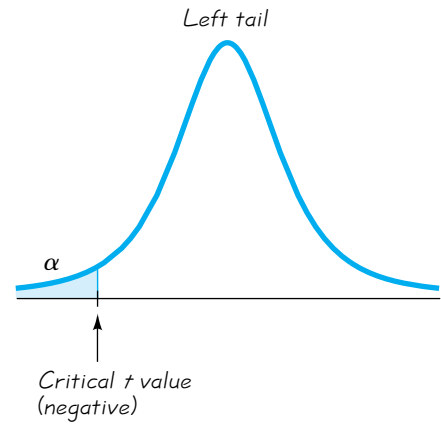
|           |   |             |            |   |   |   |       |     |    |    |       |     |    |    |       |     |    |   |  |   |   |   |       |     |    |    |       |     |    |   |
|-----------|---|-------------|------------|---|---|---|-------|-----|----|----|-------|-----|----|----|-------|-----|----|---|--|---|---|---|-------|-----|----|----|-------|-----|----|---|
| <b>1A</b> | Mathematically, define the simple kriging error variance, and express it as a function of variance-covariance function.<br><br><b>OR</b><br><br>Mathematically, define the ordinary kriging error variance, and express it as a function of variogram function.   | <b>[10]</b> | <b>CO5</b> |   |   |   |       |     |    |    |       |     |    |    |       |     |    |   |  |   |   |   |       |     |    |    |       |     |    |   |
| <b>1B</b> | Use simple kriging to estimate the value of $Z(x_0)$ at $x_0 = (180, 120)$ . Given $E[Z(x)] = 110$ and the covariance function $2000 * \exp(\frac{-h}{250})$ .<br><table border="1" data-bbox="415 688 1089 846"><tr><td></td><td>X</td><td>Y</td><td>Z</td></tr><tr><td><math>x_1</math></td><td>387</td><td>72</td><td>50</td></tr><tr><td><math>x_2</math></td><td>392</td><td>81</td><td>56</td></tr><tr><td><math>x_3</math></td><td>388</td><td>56</td><td>a</td></tr></table> here $a=52 + \frac{3}{250}d$ , where $d$ is the three digit number formed by last three digits of your roll number. For example if your roll number is R870218125, then $d = 125$ .<br><b>OR</b><br>Use ordinary kriging to estimate the value of $Z(x_0)$ at $x_0 = (180, 120)$ . Given, covariance function as $2000 * \exp(\frac{-h}{250})$ .<br><table border="1" data-bbox="415 1066 1089 1182"><tr><td></td><td>X</td><td>Y</td><td>Z</td></tr><tr><td><math>x_1</math></td><td>387</td><td>72</td><td>50</td></tr><tr><td><math>x_2</math></td><td>392</td><td>81</td><td>a</td></tr></table> here $a=55 + \frac{3}{250}d$ , where $d$ is the three digit number formed by last three digits of your roll number. For example if your roll number is R870218125, then $d = 125$ . |             |            | X | Y | Z | $x_1$ | 387 | 72 | 50 | $x_2$ | 392 | 81 | 56 | $x_3$ | 388 | 56 | a |  | X | Y | Z | $x_1$ | 387 | 72 | 50 | $x_2$ | 392 | 81 | a |
|           | X   | Y           | Z          |   |   |   |       |     |    |    |       |     |    |    |       |     |    |   |  |   |   |   |       |     |    |    |       |     |    |   |
| $x_1$     | 387   | 72          | 50         |   |   |   |       |     |    |    |       |     |    |    |       |     |    |   |  |   |   |   |       |     |    |    |       |     |    |   |
| $x_2$     | 392   | 81          | 56         |   |   |   |       |     |    |    |       |     |    |    |       |     |    |   |  |   |   |   |       |     |    |    |       |     |    |   |
| $x_3$     | 388   | 56          | a          |   |   |   |       |     |    |    |       |     |    |    |       |     |    |   |  |   |   |   |       |     |    |    |       |     |    |   |
|           | X   | Y           | Z          |   |   |   |       |     |    |    |       |     |    |    |       |     |    |   |  |   |   |   |       |     |    |    |       |     |    |   |
| $x_1$     | 387   | 72          | 50         |   |   |   |       |     |    |    |       |     |    |    |       |     |    |   |  |   |   |   |       |     |    |    |       |     |    |   |
| $x_2$     | 392   | 81          | a          |   |   |   |       |     |    |    |       |     |    |    |       |     |    |   |  |   |   |   |       |     |    |    |       |     |    |   |

**STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.**

| <b>Z</b> | <b>.00</b> | <b>.01</b> | <b>.02</b> | <b>.03</b> | <b>.04</b> | <b>.05</b> | <b>.06</b> | <b>.07</b> | <b>.08</b> | <b>.09</b> |
|----------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| -3.9     | .00005     | .00005     | .00004     | .00004     | .00004     | .00004     | .00004     | .00004     | .00003     | .00003     |
| -3.8     | .00007     | .00007     | .00007     | .00006     | .00006     | .00006     | .00006     | .00005     | .00005     | .00005     |
| -3.7     | .00011     | .00010     | .00010     | .00010     | .00009     | .00009     | .00008     | .00008     | .00008     | .00008     |
| -3.6     | .00016     | .00015     | .00015     | .00014     | .00014     | .00013     | .00013     | .00012     | .00012     | .00011     |
| -3.5     | .00023     | .00022     | .00022     | .00021     | .00020     | .00019     | .00019     | .00018     | .00017     | .00017     |
| -3.4     | .00034     | .00032     | .00031     | .00030     | .00029     | .00028     | .00027     | .00026     | .00025     | .00024     |
| -3.3     | .00048     | .00047     | .00045     | .00043     | .00042     | .00040     | .00039     | .00038     | .00036     | .00035     |
| -3.2     | .00069     | .00066     | .00064     | .00062     | .00060     | .00058     | .00056     | .00054     | .00052     | .00050     |
| -3.1     | .00097     | .00094     | .00090     | .00087     | .00084     | .00082     | .00079     | .00076     | .00074     | .00071     |
| -3.0     | .00135     | .00131     | .00126     | .00122     | .00118     | .00114     | .00111     | .00107     | .00104     | .00100     |
| -2.9     | .00187     | .00181     | .00175     | .00169     | .00164     | .00159     | .00154     | .00149     | .00144     | .00139     |
| -2.8     | .00256     | .00248     | .00240     | .00233     | .00226     | .00219     | .00212     | .00205     | .00199     | .00193     |
| -2.7     | .00347     | .00336     | .00326     | .00317     | .00307     | .00298     | .00289     | .00280     | .00272     | .00264     |
| -2.6     | .00466     | .00453     | .00440     | .00427     | .00415     | .00402     | .00391     | .00379     | .00368     | .00357     |
| -2.5     | .00621     | .00604     | .00587     | .00570     | .00554     | .00539     | .00523     | .00508     | .00494     | .00480     |
| -2.4     | .00820     | .00798     | .00776     | .00755     | .00734     | .00714     | .00695     | .00676     | .00657     | .00639     |
| -2.3     | .01072     | .01044     | .01017     | .00990     | .00964     | .00939     | .00914     | .00889     | .00866     | .00842     |
| -2.2     | .01390     | .01355     | .01321     | .01287     | .01255     | .01222     | .01191     | .01160     | .01130     | .01101     |
| -2.1     | .01786     | .01743     | .01700     | .01659     | .01618     | .01578     | .01539     | .01500     | .01463     | .01426     |
| -2.0     | .02275     | .02222     | .02169     | .02118     | .02068     | .02018     | .01970     | .01923     | .01876     | .01831     |
| -1.9     | .02872     | .02807     | .02743     | .02680     | .02619     | .02559     | .02500     | .02442     | .02385     | .02330     |
| -1.8     | .03593     | .03515     | .03438     | .03362     | .03288     | .03216     | .03144     | .03074     | .03005     | .02938     |
| -1.7     | .04457     | .04363     | .04272     | .04182     | .04093     | .04006     | .03920     | .03836     | .03754     | .03673     |
| -1.6     | .05480     | .05370     | .05262     | .05155     | .05050     | .04947     | .04846     | .04746     | .04648     | .04551     |
| -1.5     | .06681     | .06552     | .06426     | .06301     | .06178     | .06057     | .05938     | .05821     | .05705     | .05592     |
| -1.4     | .08076     | .07927     | .07780     | .07636     | .07493     | .07353     | .07215     | .07078     | .06944     | .06811     |
| -1.3     | .09680     | .09510     | .09342     | .09176     | .09012     | .08851     | .08691     | .08534     | .08379     | .08226     |
| -1.2     | .11507     | .11314     | .11123     | .10935     | .10749     | .10565     | .10383     | .10204     | .10027     | .09853     |
| -1.1     | .13567     | .13350     | .13136     | .12924     | .12714     | .12507     | .12302     | .12100     | .11900     | .11702     |
| -1.0     | .15866     | .15625     | .15386     | .15151     | .14917     | .14686     | .14457     | .14231     | .14007     | .13786     |
| -0.9     | .18406     | .18141     | .17879     | .17619     | .17361     | .17106     | .16853     | .16602     | .16354     | .16109     |
| -0.8     | .21186     | .20897     | .20611     | .20327     | .20045     | .19766     | .19489     | .19215     | .18943     | .18673     |
| -0.7     | .24196     | .23885     | .23576     | .23270     | .22965     | .22663     | .22363     | .22065     | .21770     | .21476     |
| -0.6     | .27425     | .27093     | .26763     | .26435     | .26109     | .25785     | .25463     | .25143     | .24825     | .24510     |
| -0.5     | .30854     | .30503     | .30153     | .29806     | .29460     | .29116     | .28774     | .28434     | .28096     | .27760     |
| -0.4     | .34458     | .34090     | .33724     | .33360     | .32997     | .32636     | .32276     | .31918     | .31561     | .31207     |
| -0.3     | .38209     | .37828     | .37448     | .37070     | .36693     | .36317     | .35942     | .35569     | .35197     | .34827     |
| -0.2     | .42074     | .41683     | .41294     | .40905     | .40517     | .40129     | .39743     | .39358     | .38974     | .38591     |
| -0.1     | .46017     | .45620     | .45224     | .44828     | .44433     | .44038     | .43644     | .43251     | .42858     | .42465     |
| -0.0     | .50000     | .49601     | .49202     | .48803     | .48405     | .48006     | .47608     | .47210     | .46812     | .46414     |



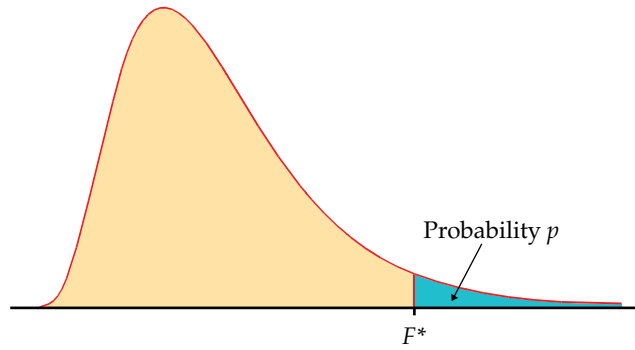
| <b>TABLE A-3</b>   |                   | <i>t</i> Distribution: Critical <i>t</i> Values |                  |       |       |  |
|--------------------|-------------------|---|------------------|-------|-------|--|
|                    | 0.005             | 0.01  | Area in One Tail |       |       |  |
|                    |                   |   | 0.025            | 0.05  | 0.10  |  |
| Degrees of Freedom | Area in Two Tails |   |                  |       |       |  |
|                    | 0.01              | 0.02  | 0.05             | 0.10  | 0.20  |  |
| 1                  | <b>63.657</b>     | 31.821  | <b>12.706</b>    | 6.314 | 3.078 |  |
| 2                  | <b>9.925</b>      | 6.965   | <b>4.303</b>     | 2.920 | 1.886 |  |
| 3                  | <b>5.841</b>      | 4.541   | <b>3.182</b>     | 2.353 | 1.638 |  |
| 4                  | <b>4.604</b>      | 3.747   | <b>2.776</b>     | 2.132 | 1.533 |  |
| 5                  | <b>4.032</b>      | 3.365   | <b>2.571</b>     | 2.015 | 1.476 |  |
| 6                  | <b>3.707</b>      | 3.143   | <b>2.447</b>     | 1.943 | 1.440 |  |
| 7                  | <b>3.499</b>      | 2.998   | <b>2.365</b>     | 1.895 | 1.415 |  |
| 8                  | <b>3.355</b>      | 2.896   | <b>2.306</b>     | 1.860 | 1.397 |  |
| 9                  | <b>3.250</b>      | 2.821   | <b>2.262</b>     | 1.833 | 1.383 |  |
| 10                 | <b>3.169</b>      | 2.764   | <b>2.228</b>     | 1.812 | 1.372 |  |
| 11                 | <b>3.106</b>      | 2.718   | <b>2.201</b>     | 1.796 | 1.363 |  |
| 12                 | <b>3.055</b>      | 2.681   | <b>2.179</b>     | 1.782 | 1.356 |  |
| 13                 | <b>3.012</b>      | 2.650   | <b>2.160</b>     | 1.771 | 1.350 |  |
| 14                 | <b>2.977</b>      | 2.624   | <b>2.145</b>     | 1.761 | 1.345 |  |
| 15                 | <b>2.947</b>      | 2.602   | <b>2.131</b>     | 1.753 | 1.341 |  |
| 16                 | <b>2.921</b>      | 2.583   | <b>2.120</b>     | 1.746 | 1.337 |  |
| 17                 | <b>2.898</b>      | 2.567   | <b>2.110</b>     | 1.740 | 1.333 |  |
| 18                 | <b>2.878</b>      | 2.552   | <b>2.101</b>     | 1.734 | 1.330 |  |
| 19                 | <b>2.861</b>      | 2.539   | <b>2.093</b>     | 1.729 | 1.328 |  |
| 20                 | <b>2.845</b>      | 2.528   | <b>2.086</b>     | 1.725 | 1.325 |  |
| 21                 | <b>2.831</b>      | 2.518   | <b>2.080</b>     | 1.721 | 1.323 |  |
| 22                 | <b>2.819</b>      | 2.508   | <b>2.074</b>     | 1.717 | 1.321 |  |
| 23                 | <b>2.807</b>      | 2.500   | <b>2.069</b>     | 1.714 | 1.319 |  |
| 24                 | <b>2.797</b>      | 2.492   | <b>2.064</b>     | 1.711 | 1.318 |  |
| 25                 | <b>2.787</b>      | 2.485   | <b>2.060</b>     | 1.708 | 1.316 |  |
| 26                 | <b>2.779</b>      | 2.479   | <b>2.056</b>     | 1.706 | 1.315 |  |
| 27                 | <b>2.771</b>      | 2.473   | <b>2.052</b>     | 1.703 | 1.314 |  |
| 28                 | <b>2.763</b>      | 2.467   | <b>2.048</b>     | 1.701 | 1.313 |  |
| 29                 | <b>2.756</b>      | 2.462   | <b>2.045</b>     | 1.699 | 1.311 |  |
| 30                 | <b>2.750</b>      | 2.457   | <b>2.042</b>     | 1.697 | 1.310 |  |
| 31                 | <b>2.744</b>      | 2.453   | <b>2.040</b>     | 1.696 | 1.309 |  |
| 32                 | <b>2.738</b>      | 2.449   | <b>2.037</b>     | 1.694 | 1.309 |  |
| 34                 | <b>2.728</b>      | 2.441   | <b>2.032</b>     | 1.691 | 1.307 |  |
| 36                 | <b>2.719</b>      | 2.434   | <b>2.028</b>     | 1.688 | 1.306 |  |
| 38                 | <b>2.712</b>      | 2.429   | <b>2.024</b>     | 1.686 | 1.304 |  |
| 40                 | <b>2.704</b>      | 2.423   | <b>2.021</b>     | 1.684 | 1.303 |  |
| 45                 | <b>2.690</b>      | 2.412   | <b>2.014</b>     | 1.679 | 1.301 |  |
| 50                 | <b>2.678</b>      | 2.403   | <b>2.009</b>     | 1.676 | 1.299 |  |
| 55                 | <b>2.668</b>      | 2.396   | <b>2.004</b>     | 1.673 | 1.297 |  |
| 60                 | <b>2.660</b>      | 2.390   | <b>2.000</b>     | 1.671 | 1.296 |  |
| 65                 | <b>2.654</b>      | 2.385   | <b>1.997</b>     | 1.669 | 1.295 |  |
| 70                 | <b>2.648</b>      | 2.381   | <b>1.994</b>     | 1.667 | 1.294 |  |
| 75                 | <b>2.643</b>      | 2.377   | <b>1.992</b>     | 1.665 | 1.293 |  |
| 80                 | <b>2.639</b>      | 2.374   | <b>1.990</b>     | 1.664 | 1.292 |  |
| 90                 | <b>2.632</b>      | 2.368   | <b>1.987</b>     | 1.662 | 1.291 |  |
| 100                | <b>2.626</b>      | 2.364   | <b>1.984</b>     | 1.660 | 1.290 |  |
| 200                | <b>2.601</b>      | 2.345   | <b>1.972</b>     | 1.653 | 1.286 |  |
| 300                | <b>2.592</b>      | 2.339   | <b>1.968</b>     | 1.650 | 1.284 |  |
| 400                | <b>2.588</b>      | 2.336   | <b>1.966</b>     | 1.649 | 1.284 |  |
| 500                | <b>2.586</b>      | 2.334   | <b>1.965</b>     | 1.648 | 1.283 |  |
| 750                | <b>2.582</b>      | 2.331   | <b>1.963</b>     | 1.647 | 1.283 |  |
| 1000               | <b>2.581</b>      | 2.330   | <b>1.962</b>     | 1.646 | 1.282 |  |
| 2000               | <b>2.578</b>      | 2.328   | <b>1.961</b>     | 1.646 | 1.282 |  |
| Large              | <b>2.576</b>      | 2.326   | <b>1.960</b>     | 1.645 | 1.282 |  |



**Chi-square Distribution Table**

| d.f. | .995  | .99   | .975  | .95   | .9    | .1     | .05    | .025   | .01    |
|------|-------|-------|-------|-------|-------|--------|--------|--------|--------|
| 1    | 0.00  | 0.00  | 0.00  | 0.00  | 0.02  | 2.71   | 3.84   | 5.02   | 6.63   |
| 2    | 0.01  | 0.02  | 0.05  | 0.10  | 0.21  | 4.61   | 5.99   | 7.38   | 9.21   |
| 3    | 0.07  | 0.11  | 0.22  | 0.35  | 0.58  | 6.25   | 7.81   | 9.35   | 11.34  |
| 4    | 0.21  | 0.30  | 0.48  | 0.71  | 1.06  | 7.78   | 9.49   | 11.14  | 13.28  |
| 5    | 0.41  | 0.55  | 0.83  | 1.15  | 1.61  | 9.24   | 11.07  | 12.83  | 15.09  |
| 6    | 0.68  | 0.87  | 1.24  | 1.64  | 2.20  | 10.64  | 12.59  | 14.45  | 16.81  |
| 7    | 0.99  | 1.24  | 1.69  | 2.17  | 2.83  | 12.02  | 14.07  | 16.01  | 18.48  |
| 8    | 1.34  | 1.65  | 2.18  | 2.73  | 3.49  | 13.36  | 15.51  | 17.53  | 20.09  |
| 9    | 1.73  | 2.09  | 2.70  | 3.33  | 4.17  | 14.68  | 16.92  | 19.02  | 21.67  |
| 10   | 2.16  | 2.56  | 3.25  | 3.94  | 4.87  | 15.99  | 18.31  | 20.48  | 23.21  |
| 11   | 2.60  | 3.05  | 3.82  | 4.57  | 5.58  | 17.28  | 19.68  | 21.92  | 24.72  |
| 12   | 3.07  | 3.57  | 4.40  | 5.23  | 6.30  | 18.55  | 21.03  | 23.34  | 26.22  |
| 13   | 3.57  | 4.11  | 5.01  | 5.89  | 7.04  | 19.81  | 22.36  | 24.74  | 27.69  |
| 14   | 4.07  | 4.66  | 5.63  | 6.57  | 7.79  | 21.06  | 23.68  | 26.12  | 29.14  |
| 15   | 4.60  | 5.23  | 6.26  | 7.26  | 8.55  | 22.31  | 25.00  | 27.49  | 30.58  |
| 16   | 5.14  | 5.81  | 6.91  | 7.96  | 9.31  | 23.54  | 26.30  | 28.85  | 32.00  |
| 17   | 5.70  | 6.41  | 7.56  | 8.67  | 10.09 | 24.77  | 27.59  | 30.19  | 33.41  |
| 18   | 6.26  | 7.01  | 8.23  | 9.39  | 10.86 | 25.99  | 28.87  | 31.53  | 34.81  |
| 19   | 6.84  | 7.63  | 8.91  | 10.12 | 11.65 | 27.20  | 30.14  | 32.85  | 36.19  |
| 20   | 7.43  | 8.26  | 9.59  | 10.85 | 12.44 | 28.41  | 31.41  | 34.17  | 37.57  |
| 22   | 8.64  | 9.54  | 10.98 | 12.34 | 14.04 | 30.81  | 33.92  | 36.78  | 40.29  |
| 24   | 9.89  | 10.86 | 12.40 | 13.85 | 15.66 | 33.20  | 36.42  | 39.36  | 42.98  |
| 26   | 11.16 | 12.20 | 13.84 | 15.38 | 17.29 | 35.56  | 38.89  | 41.92  | 45.64  |
| 28   | 12.46 | 13.56 | 15.31 | 16.93 | 18.94 | 37.92  | 41.34  | 44.46  | 48.28  |
| 30   | 13.79 | 14.95 | 16.79 | 18.49 | 20.60 | 40.26  | 43.77  | 46.98  | 50.89  |
| 32   | 15.13 | 16.36 | 18.29 | 20.07 | 22.27 | 42.58  | 46.19  | 49.48  | 53.49  |
| 34   | 16.50 | 17.79 | 19.81 | 21.66 | 23.95 | 44.90  | 48.60  | 51.97  | 56.06  |
| 38   | 19.29 | 20.69 | 22.88 | 24.88 | 27.34 | 49.51  | 53.38  | 56.90  | 61.16  |
| 42   | 22.14 | 23.65 | 26.00 | 28.14 | 30.77 | 54.09  | 58.12  | 61.78  | 66.21  |
| 46   | 25.04 | 26.66 | 29.16 | 31.44 | 34.22 | 58.64  | 62.83  | 66.62  | 71.20  |
| 50   | 27.99 | 29.71 | 32.36 | 34.76 | 37.69 | 63.17  | 67.50  | 71.42  | 76.15  |
| 55   | 31.73 | 33.57 | 36.40 | 38.96 | 42.06 | 68.80  | 73.31  | 77.38  | 82.29  |
| 60   | 35.53 | 37.48 | 40.48 | 43.19 | 46.46 | 74.40  | 79.08  | 83.30  | 88.38  |
| 65   | 39.38 | 41.44 | 44.60 | 47.45 | 50.88 | 79.97  | 84.82  | 89.18  | 94.42  |
| 70   | 43.28 | 45.44 | 48.76 | 51.74 | 55.33 | 85.53  | 90.53  | 95.02  | 100.43 |
| 75   | 47.21 | 49.48 | 52.94 | 56.05 | 59.79 | 91.06  | 96.22  | 100.84 | 106.39 |
| 80   | 51.17 | 53.54 | 57.15 | 60.39 | 64.28 | 96.58  | 101.88 | 106.63 | 112.33 |
| 85   | 55.17 | 57.63 | 61.39 | 64.75 | 68.78 | 102.08 | 107.52 | 112.39 | 118.24 |
| 90   | 59.20 | 61.75 | 65.65 | 69.13 | 73.29 | 107.57 | 113.15 | 118.14 | 124.12 |
| 95   | 63.25 | 65.90 | 69.92 | 73.52 | 77.82 | 113.04 | 118.75 | 123.86 | 129.97 |
| 100  | 67.33 | 70.06 | 74.22 | 77.93 | 82.36 | 118.50 | 124.34 | 129.56 | 135.81 |

Table entry for  $p$  is the critical value  $F^*$  with probability  $p$  lying to its right.



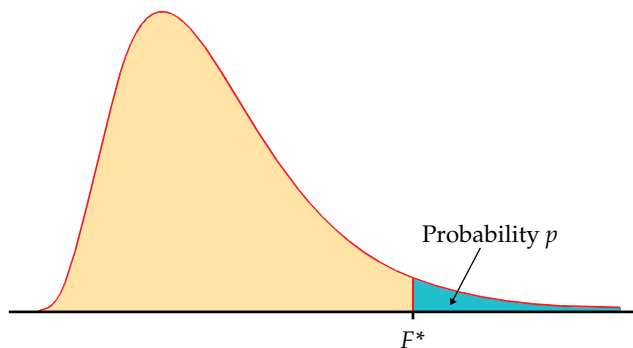
**TABLE E**

**F critical values**

|                                       |      | Degrees of freedom in the numerator |        |        |        |        |        |        |        |        |        |
|---------------------------------------|------|-------------------------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $p$                                   |      | 1                                   | 2      | 3      | 4      | 5      | 6      | 7      | 8      | 9      |        |
| Degrees of freedom in the denominator | 1    | .100                                | 39.86  | 49.50  | 53.59  | 55.83  | 57.24  | 58.20  | 58.91  | 59.44  | 59.86  |
|                                       |      | .050                                | 161.45 | 199.50 | 215.71 | 224.58 | 230.16 | 233.99 | 236.77 | 238.88 | 240.54 |
|                                       |      | .025                                | 647.79 | 799.50 | 864.16 | 899.58 | 921.85 | 937.11 | 948.22 | 956.66 | 963.28 |
|                                       |      | .010                                | 4052.2 | 4999.5 | 5403.4 | 5624.6 | 5763.6 | 5859.0 | 5928.4 | 5981.1 | 6022.5 |
|                                       |      | .001                                | 405284 | 500000 | 540379 | 562500 | 576405 | 585937 | 592873 | 598144 | 602284 |
|                                       | 2    | .100                                | 8.53   | 9.00   | 9.16   | 9.24   | 9.29   | 9.33   | 9.35   | 9.37   | 9.38   |
|                                       |      | .050                                | 18.51  | 19.00  | 19.16  | 19.25  | 19.30  | 19.33  | 19.35  | 19.37  | 19.38  |
|                                       |      | .025                                | 38.51  | 39.00  | 39.17  | 39.25  | 39.30  | 39.33  | 39.36  | 39.37  | 39.39  |
|                                       |      | .010                                | 98.50  | 99.00  | 99.17  | 99.25  | 99.30  | 99.33  | 99.36  | 99.37  | 99.39  |
|                                       |      | .001                                | 998.50 | 999.00 | 999.17 | 999.25 | 999.30 | 999.33 | 999.36 | 999.37 | 999.39 |
|                                       | 3    | .100                                | 5.54   | 5.46   | 5.39   | 5.34   | 5.31   | 5.28   | 5.27   | 5.25   | 5.24   |
|                                       |      | .050                                | 10.13  | 9.55   | 9.28   | 9.12   | 9.01   | 8.94   | 8.89   | 8.85   | 8.81   |
|                                       |      | .025                                | 17.44  | 16.04  | 15.44  | 15.10  | 14.88  | 14.73  | 14.62  | 14.54  | 14.47  |
|                                       |      | .010                                | 34.12  | 30.82  | 29.46  | 28.71  | 28.24  | 27.91  | 27.67  | 27.49  | 27.35  |
|                                       |      | .001                                | 167.03 | 148.50 | 141.11 | 137.10 | 134.58 | 132.85 | 131.58 | 130.62 | 129.86 |
|                                       | 4    | .100                                | 4.54   | 4.32   | 4.19   | 4.11   | 4.05   | 4.01   | 3.98   | 3.95   | 3.94   |
|                                       |      | .050                                | 7.71   | 6.94   | 6.59   | 6.39   | 6.26   | 6.16   | 6.09   | 6.04   | 6.00   |
|                                       |      | .025                                | 12.22  | 10.65  | 9.98   | 9.60   | 9.36   | 9.20   | 9.07   | 8.98   | 8.90   |
|                                       |      | .010                                | 21.20  | 18.00  | 16.69  | 15.98  | 15.52  | 15.21  | 14.98  | 14.80  | 14.66  |
|                                       |      | .001                                | 74.14  | 61.25  | 56.18  | 53.44  | 51.71  | 50.53  | 49.66  | 49.00  | 48.47  |
| 5                                     | .100 | 4.06                                | 3.78   | 3.62   | 3.52   | 3.45   | 3.40   | 3.37   | 3.34   | 3.32   |        |
|                                       | .050 | 6.61                                | 5.79   | 5.41   | 5.19   | 5.05   | 4.95   | 4.88   | 4.82   | 4.77   |        |
|                                       | .025 | 10.01                               | 8.43   | 7.76   | 7.39   | 7.15   | 6.98   | 6.85   | 6.76   | 6.68   |        |
|                                       | .010 | 16.26                               | 13.27  | 12.06  | 11.39  | 10.97  | 10.67  | 10.46  | 10.29  | 10.16  |        |
|                                       | .001 | 47.18                               | 37.12  | 33.20  | 31.09  | 29.75  | 28.83  | 28.16  | 27.65  | 27.24  |        |
| 6                                     | .100 | 3.78                                | 3.46   | 3.29   | 3.18   | 3.11   | 3.05   | 3.01   | 2.98   | 2.96   |        |
|                                       | .050 | 5.99                                | 5.14   | 4.76   | 4.53   | 4.39   | 4.28   | 4.21   | 4.15   | 4.10   |        |
|                                       | .025 | 8.81                                | 7.26   | 6.60   | 6.23   | 5.99   | 5.82   | 5.70   | 5.60   | 5.52   |        |
|                                       | .010 | 13.75                               | 10.92  | 9.78   | 9.15   | 8.75   | 8.47   | 8.26   | 8.10   | 7.98   |        |
|                                       | .001 | 35.51                               | 27.00  | 23.70  | 21.92  | 20.80  | 20.03  | 19.46  | 19.03  | 18.69  |        |
| 7                                     | .100 | 3.59                                | 3.26   | 3.07   | 2.96   | 2.88   | 2.83   | 2.78   | 2.75   | 2.72   |        |
|                                       | .050 | 5.59                                | 4.74   | 4.35   | 4.12   | 3.97   | 3.87   | 3.79   | 3.73   | 3.68   |        |
|                                       | .025 | 8.07                                | 6.54   | 5.89   | 5.52   | 5.29   | 5.12   | 4.99   | 4.90   | 4.82   |        |
|                                       | .010 | 12.25                               | 9.55   | 8.45   | 7.85   | 7.46   | 7.19   | 6.99   | 6.84   | 6.72   |        |
|                                       | .001 | 29.25                               | 21.69  | 18.77  | 17.20  | 16.21  | 15.52  | 15.02  | 14.63  | 14.33  |        |



Table entry for  $p$  is the critical value  $F^*$  with probability  $p$  lying to its right.



**TABLE E**

**F critical values (continued)**

| Degrees of freedom in the numerator |        |        |        |        |        |        |        |        |        |        |
|-------------------------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 10                                  | 12     | 15     | 20     | 25     | 30     | 40     | 50     | 60     | 120    | 1000   |
| 60.19                               | 60.71  | 61.22  | 61.74  | 62.05  | 62.26  | 62.53  | 62.69  | 62.79  | 63.06  | 63.30  |
| 241.88                              | 243.91 | 245.95 | 248.01 | 249.26 | 250.10 | 251.14 | 251.77 | 252.20 | 253.25 | 254.19 |
| 968.63                              | 976.71 | 984.87 | 993.10 | 998.08 | 1001.4 | 1005.6 | 1008.1 | 1009.8 | 1014.0 | 1017.7 |
| 6055.8                              | 6106.3 | 6157.3 | 6208.7 | 6239.8 | 6260.6 | 6286.8 | 6302.5 | 6313.0 | 6339.4 | 6362.7 |
| 605621                              | 610668 | 615764 | 620908 | 624017 | 626099 | 628712 | 630285 | 631337 | 633972 | 636301 |
| 9.39                                | 9.41   | 9.42   | 9.44   | 9.45   | 9.46   | 9.47   | 9.47   | 9.47   | 9.48   | 9.49   |
| 19.40                               | 19.41  | 19.43  | 19.45  | 19.46  | 19.46  | 19.47  | 19.48  | 19.48  | 19.49  | 19.49  |
| 39.40                               | 39.41  | 39.43  | 39.45  | 39.46  | 39.46  | 39.47  | 39.48  | 39.48  | 39.49  | 39.50  |
| 99.40                               | 99.42  | 99.43  | 99.45  | 99.46  | 99.47  | 99.47  | 99.48  | 99.48  | 99.49  | 99.50  |
| 999.40                              | 999.42 | 999.43 | 999.45 | 999.46 | 999.47 | 999.47 | 999.48 | 999.48 | 999.49 | 999.50 |
| 5.23                                | 5.22   | 5.20   | 5.18   | 5.17   | 5.17   | 5.16   | 5.15   | 5.15   | 5.14   | 5.13   |
| 8.79                                | 8.74   | 8.70   | 8.66   | 8.63   | 8.62   | 8.59   | 8.58   | 8.57   | 8.55   | 8.53   |
| 14.42                               | 14.34  | 14.25  | 14.17  | 14.12  | 14.08  | 14.04  | 14.01  | 13.99  | 13.95  | 13.91  |
| 27.23                               | 27.05  | 26.87  | 26.69  | 26.58  | 26.50  | 26.41  | 26.35  | 26.32  | 26.22  | 26.14  |
| 129.25                              | 128.32 | 127.37 | 126.42 | 125.84 | 125.45 | 124.96 | 124.66 | 124.47 | 123.97 | 123.53 |
| 3.92                                | 3.90   | 3.87   | 3.84   | 3.83   | 3.82   | 3.80   | 3.80   | 3.79   | 3.78   | 3.76   |
| 5.96                                | 5.91   | 5.86   | 5.80   | 5.77   | 5.75   | 5.72   | 5.70   | 5.69   | 5.66   | 5.63   |
| 8.84                                | 8.75   | 8.66   | 8.56   | 8.50   | 8.46   | 8.41   | 8.38   | 8.36   | 8.31   | 8.26   |
| 14.55                               | 14.37  | 14.20  | 14.02  | 13.91  | 13.84  | 13.75  | 13.69  | 13.65  | 13.56  | 13.47  |
| 48.05                               | 47.41  | 46.76  | 46.10  | 45.70  | 45.43  | 45.09  | 44.88  | 44.75  | 44.40  | 44.09  |
| 3.30                                | 3.27   | 3.24   | 3.21   | 3.19   | 3.17   | 3.16   | 3.15   | 3.14   | 3.12   | 3.11   |
| 4.74                                | 4.68   | 4.62   | 4.56   | 4.52   | 4.50   | 4.46   | 4.44   | 4.43   | 4.40   | 4.37   |
| 6.62                                | 6.52   | 6.43   | 6.33   | 6.27   | 6.23   | 6.18   | 6.14   | 6.12   | 6.07   | 6.02   |
| 10.05                               | 9.89   | 9.72   | 9.55   | 9.45   | 9.38   | 9.29   | 9.24   | 9.20   | 9.11   | 9.03   |
| 26.92                               | 26.42  | 25.91  | 25.39  | 25.08  | 24.87  | 24.60  | 24.44  | 24.33  | 24.06  | 23.82  |
| 2.94                                | 2.90   | 2.87   | 2.84   | 2.81   | 2.80   | 2.78   | 2.77   | 2.76   | 2.74   | 2.72   |
| 4.06                                | 4.00   | 3.94   | 3.87   | 3.83   | 3.81   | 3.77   | 3.75   | 3.74   | 3.70   | 3.67   |
| 5.46                                | 5.37   | 5.27   | 5.17   | 5.11   | 5.07   | 5.01   | 4.98   | 4.96   | 4.90   | 4.86   |
| 7.87                                | 7.72   | 7.56   | 7.40   | 7.30   | 7.23   | 7.14   | 7.09   | 7.06   | 6.97   | 6.89   |
| 18.41                               | 17.99  | 17.56  | 17.12  | 16.85  | 16.67  | 16.44  | 16.31  | 16.21  | 15.98  | 15.77  |
| 2.70                                | 2.67   | 2.63   | 2.59   | 2.57   | 2.56   | 2.54   | 2.52   | 2.51   | 2.49   | 2.47   |
| 3.64                                | 3.57   | 3.51   | 3.44   | 3.40   | 3.38   | 3.34   | 3.32   | 3.30   | 3.27   | 3.23   |
| 4.76                                | 4.67   | 4.57   | 4.47   | 4.40   | 4.36   | 4.31   | 4.28   | 4.25   | 4.20   | 4.15   |
| 6.62                                | 6.47   | 6.31   | 6.16   | 6.06   | 5.99   | 5.91   | 5.86   | 5.82   | 5.74   | 5.66   |
| 14.08                               | 13.71  | 13.32  | 12.93  | 12.69  | 12.53  | 12.33  | 12.20  | 12.12  | 11.91  | 11.72  |

(Continued)

**TABLE E**

*F* critical values (continued)

|                                       |    | Degrees of freedom in the numerator |       |       |       |       |       |       |       |       |       |       |
|---------------------------------------|----|-------------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| <i>p</i>                              |    | 1                                   | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     |       |       |
| Degrees of freedom in the denominator | 8  | .100                                | 3.46  | 3.11  | 2.92  | 2.81  | 2.73  | 2.67  | 2.62  | 2.59  | 2.56  |       |
|                                       |    | .050                                | 5.32  | 4.46  | 4.07  | 3.84  | 3.69  | 3.58  | 3.50  | 3.44  | 3.39  |       |
|                                       |    | .025                                | 7.57  | 6.06  | 5.42  | 5.05  | 4.82  | 4.65  | 4.53  | 4.43  | 4.36  |       |
|                                       |    | .010                                | 11.26 | 8.65  | 7.59  | 7.01  | 6.63  | 6.37  | 6.18  | 6.03  | 5.91  |       |
|                                       |    | .001                                | 25.41 | 18.49 | 15.83 | 14.39 | 13.48 | 12.86 | 12.40 | 12.05 | 11.77 |       |
|                                       |    | 9                                   | .100  | 3.36  | 3.01  | 2.81  | 2.69  | 2.61  | 2.55  | 2.51  | 2.47  | 2.44  |
|                                       |    |                                     | .050  | 5.12  | 4.26  | 3.86  | 3.63  | 3.48  | 3.37  | 3.29  | 3.23  | 3.18  |
|                                       |    |                                     | .025  | 7.21  | 5.71  | 5.08  | 4.72  | 4.48  | 4.32  | 4.20  | 4.10  | 4.03  |
|                                       |    |                                     | .010  | 10.56 | 8.02  | 6.99  | 6.42  | 6.06  | 5.80  | 5.61  | 5.47  | 5.35  |
|                                       |    |                                     | .001  | 22.86 | 16.39 | 13.90 | 12.56 | 11.71 | 11.13 | 10.70 | 10.37 | 10.11 |
|                                       |    | 10                                  | .100  | 3.29  | 2.92  | 2.73  | 2.61  | 2.52  | 2.46  | 2.41  | 2.38  | 2.35  |
|                                       |    |                                     | .050  | 4.96  | 4.10  | 3.71  | 3.48  | 3.33  | 3.22  | 3.14  | 3.07  | 3.02  |
|                                       |    |                                     | .025  | 6.94  | 5.46  | 4.83  | 4.47  | 4.24  | 4.07  | 3.95  | 3.85  | 3.78  |
|                                       |    |                                     | .010  | 10.04 | 7.56  | 6.55  | 5.99  | 5.64  | 5.39  | 5.20  | 5.06  | 4.94  |
|                                       |    |                                     | .001  | 21.04 | 14.91 | 12.55 | 11.28 | 10.48 | 9.93  | 9.52  | 9.20  | 8.96  |
|                                       |    | 11                                  | .100  | 3.23  | 2.86  | 2.66  | 2.54  | 2.45  | 2.39  | 2.34  | 2.30  | 2.27  |
|                                       |    |                                     | .050  | 4.84  | 3.98  | 3.59  | 3.36  | 3.20  | 3.09  | 3.01  | 2.95  | 2.90  |
|                                       |    |                                     | .025  | 6.72  | 5.26  | 4.63  | 4.28  | 4.04  | 3.88  | 3.76  | 3.66  | 3.59  |
|                                       |    |                                     | .010  | 9.65  | 7.21  | 6.22  | 5.67  | 5.32  | 5.07  | 4.89  | 4.74  | 4.63  |
|                                       |    |                                     | .001  | 19.69 | 13.81 | 11.56 | 10.35 | 9.58  | 9.05  | 8.66  | 8.35  | 8.12  |
|                                       | 12 | .100                                | 3.18  | 2.81  | 2.61  | 2.48  | 2.39  | 2.33  | 2.28  | 2.24  | 2.21  |       |
|                                       |    | .050                                | 4.75  | 3.89  | 3.49  | 3.26  | 3.11  | 3.00  | 2.91  | 2.85  | 2.80  |       |
|                                       |    | .025                                | 6.55  | 5.10  | 4.47  | 4.12  | 3.89  | 3.73  | 3.61  | 3.51  | 3.44  |       |
|                                       |    | .010                                | 9.33  | 6.93  | 5.95  | 5.41  | 5.06  | 4.82  | 4.64  | 4.50  | 4.39  |       |
|                                       |    | .001                                | 18.64 | 12.97 | 10.80 | 9.63  | 8.89  | 8.38  | 8.00  | 7.71  | 7.48  |       |
|                                       | 13 | .100                                | 3.14  | 2.76  | 2.56  | 2.43  | 2.35  | 2.28  | 2.23  | 2.20  | 2.16  |       |
|                                       |    | .050                                | 4.67  | 3.81  | 3.41  | 3.18  | 3.03  | 2.92  | 2.83  | 2.77  | 2.71  |       |
|                                       |    | .025                                | 6.41  | 4.97  | 4.35  | 4.00  | 3.77  | 3.60  | 3.48  | 3.39  | 3.31  |       |
|                                       |    | .010                                | 9.07  | 6.70  | 5.74  | 5.21  | 4.86  | 4.62  | 4.44  | 4.30  | 4.19  |       |
|                                       |    | .001                                | 17.82 | 12.31 | 10.21 | 9.07  | 8.35  | 7.86  | 7.49  | 7.21  | 6.98  |       |
|                                       | 14 | .100                                | 3.10  | 2.73  | 2.52  | 2.39  | 2.31  | 2.24  | 2.19  | 2.15  | 2.12  |       |
|                                       |    | .050                                | 4.60  | 3.74  | 3.34  | 3.11  | 2.96  | 2.85  | 2.76  | 2.70  | 2.65  |       |
|                                       |    | .025                                | 6.30  | 4.86  | 4.24  | 3.89  | 3.66  | 3.50  | 3.38  | 3.29  | 3.21  |       |
|                                       |    | .010                                | 8.86  | 6.51  | 5.56  | 5.04  | 4.69  | 4.46  | 4.28  | 4.14  | 4.03  |       |
|                                       |    | .001                                | 17.14 | 11.78 | 9.73  | 8.62  | 7.92  | 7.44  | 7.08  | 6.80  | 6.58  |       |
|                                       | 15 | .100                                | 3.07  | 2.70  | 2.49  | 2.36  | 2.27  | 2.21  | 2.16  | 2.12  | 2.09  |       |
|                                       |    | .050                                | 4.54  | 3.68  | 3.29  | 3.06  | 2.90  | 2.79  | 2.71  | 2.64  | 2.59  |       |
|                                       |    | .025                                | 6.20  | 4.77  | 4.15  | 3.80  | 3.58  | 3.41  | 3.29  | 3.20  | 3.12  |       |
|                                       |    | .010                                | 8.68  | 6.36  | 5.42  | 4.89  | 4.56  | 4.32  | 4.14  | 4.00  | 3.89  |       |
|                                       |    | .001                                | 16.59 | 11.34 | 9.34  | 8.25  | 7.57  | 7.09  | 6.74  | 6.47  | 6.26  |       |
|                                       | 16 | .100                                | 3.05  | 2.67  | 2.46  | 2.33  | 2.24  | 2.18  | 2.13  | 2.09  | 2.06  |       |
|                                       |    | .050                                | 4.49  | 3.63  | 3.24  | 3.01  | 2.85  | 2.74  | 2.66  | 2.59  | 2.54  |       |
|                                       |    | .025                                | 6.12  | 4.69  | 4.08  | 3.73  | 3.50  | 3.34  | 3.22  | 3.12  | 3.05  |       |
|                                       |    | .010                                | 8.53  | 6.23  | 5.29  | 4.77  | 4.44  | 4.20  | 4.03  | 3.89  | 3.78  |       |
|                                       |    | .001                                | 16.12 | 10.97 | 9.01  | 7.94  | 7.27  | 6.80  | 6.46  | 6.19  | 5.98  |       |
|                                       | 17 | .100                                | 3.03  | 2.64  | 2.44  | 2.31  | 2.22  | 2.15  | 2.10  | 2.06  | 2.03  |       |
|                                       |    | .050                                | 4.45  | 3.59  | 3.20  | 2.96  | 2.81  | 2.70  | 2.61  | 2.55  | 2.49  |       |
|                                       |    | .025                                | 6.04  | 4.62  | 4.01  | 3.66  | 3.44  | 3.28  | 3.16  | 3.06  | 2.98  |       |
|                                       |    | .010                                | 8.40  | 6.11  | 5.19  | 4.67  | 4.34  | 4.10  | 3.93  | 3.79  | 3.68  |       |
|                                       |    | .001                                | 15.72 | 10.66 | 8.73  | 7.68  | 7.02  | 6.56  | 6.22  | 5.96  | 5.75  |       |

**TABLE E****F critical values (continued)**

| Degrees of freedom in the numerator |       |       |       |       |       |      |      |      |      |      |
|-------------------------------------|-------|-------|-------|-------|-------|------|------|------|------|------|
| 10                                  | 12    | 15    | 20    | 25    | 30    | 40   | 50   | 60   | 120  | 1000 |
| 2.54                                | 2.50  | 2.46  | 2.42  | 2.40  | 2.38  | 2.36 | 2.35 | 2.34 | 2.32 | 2.30 |
| 3.35                                | 3.28  | 3.22  | 3.15  | 3.11  | 3.08  | 3.04 | 3.02 | 3.01 | 2.97 | 2.93 |
| 4.30                                | 4.20  | 4.10  | 4.00  | 3.94  | 3.89  | 3.84 | 3.81 | 3.78 | 3.73 | 3.68 |
| 5.81                                | 5.67  | 5.52  | 5.36  | 5.26  | 5.20  | 5.12 | 5.07 | 5.03 | 4.95 | 4.87 |
| 11.54                               | 11.19 | 10.84 | 10.48 | 10.26 | 10.11 | 9.92 | 9.80 | 9.73 | 9.53 | 9.36 |
| 2.42                                | 2.38  | 2.34  | 2.30  | 2.27  | 2.25  | 2.23 | 2.22 | 2.21 | 2.18 | 2.16 |
| 3.14                                | 3.07  | 3.01  | 2.94  | 2.89  | 2.86  | 2.83 | 2.80 | 2.79 | 2.75 | 2.71 |
| 3.96                                | 3.87  | 3.77  | 3.67  | 3.60  | 3.56  | 3.51 | 3.47 | 3.45 | 3.39 | 3.34 |
| 5.26                                | 5.11  | 4.96  | 4.81  | 4.71  | 4.65  | 4.57 | 4.52 | 4.48 | 4.40 | 4.32 |
| 9.89                                | 9.57  | 9.24  | 8.90  | 8.69  | 8.55  | 8.37 | 8.26 | 8.19 | 8.00 | 7.84 |
| 2.32                                | 2.28  | 2.24  | 2.20  | 2.17  | 2.16  | 2.13 | 2.12 | 2.11 | 2.08 | 2.06 |
| 2.98                                | 2.91  | 2.85  | 2.77  | 2.73  | 2.70  | 2.66 | 2.64 | 2.62 | 2.58 | 2.54 |
| 3.72                                | 3.62  | 3.52  | 3.42  | 3.35  | 3.31  | 3.26 | 3.22 | 3.20 | 3.14 | 3.09 |
| 4.85                                | 4.71  | 4.56  | 4.41  | 4.31  | 4.25  | 4.17 | 4.12 | 4.08 | 4.00 | 3.92 |
| 8.75                                | 8.45  | 8.13  | 7.80  | 7.60  | 7.47  | 7.30 | 7.19 | 7.12 | 6.94 | 6.78 |
| 2.25                                | 2.21  | 2.17  | 2.12  | 2.10  | 2.08  | 2.05 | 2.04 | 2.03 | 2.00 | 1.98 |
| 2.85                                | 2.79  | 2.72  | 2.65  | 2.60  | 2.57  | 2.53 | 2.51 | 2.49 | 2.45 | 2.41 |
| 3.53                                | 3.43  | 3.33  | 3.23  | 3.16  | 3.12  | 3.06 | 3.03 | 3.00 | 2.94 | 2.89 |
| 4.54                                | 4.40  | 4.25  | 4.10  | 4.01  | 3.94  | 3.86 | 3.81 | 3.78 | 3.69 | 3.61 |
| 7.92                                | 7.63  | 7.32  | 7.01  | 6.81  | 6.68  | 6.52 | 6.42 | 6.35 | 6.18 | 6.02 |
| 2.19                                | 2.15  | 2.10  | 2.06  | 2.03  | 2.01  | 1.99 | 1.97 | 1.96 | 1.93 | 1.91 |
| 2.75                                | 2.69  | 2.62  | 2.54  | 2.50  | 2.47  | 2.43 | 2.40 | 2.38 | 2.34 | 2.30 |
| 3.37                                | 3.28  | 3.18  | 3.07  | 3.01  | 2.96  | 2.91 | 2.87 | 2.85 | 2.79 | 2.73 |
| 4.30                                | 4.16  | 4.01  | 3.86  | 3.76  | 3.70  | 3.62 | 3.57 | 3.54 | 3.45 | 3.37 |
| 7.29                                | 7.00  | 6.71  | 6.40  | 6.22  | 6.09  | 5.93 | 5.83 | 5.76 | 5.59 | 5.44 |
| 2.14                                | 2.10  | 2.05  | 2.01  | 1.98  | 1.96  | 1.93 | 1.92 | 1.90 | 1.88 | 1.85 |
| 2.67                                | 2.60  | 2.53  | 2.46  | 2.41  | 2.38  | 2.34 | 2.31 | 2.30 | 2.25 | 2.21 |
| 3.25                                | 3.15  | 3.05  | 2.95  | 2.88  | 2.84  | 2.78 | 2.74 | 2.72 | 2.66 | 2.60 |
| 4.10                                | 3.96  | 3.82  | 3.66  | 3.57  | 3.51  | 3.43 | 3.38 | 3.34 | 3.25 | 3.18 |
| 6.80                                | 6.52  | 6.23  | 5.93  | 5.75  | 5.63  | 5.47 | 5.37 | 5.30 | 5.14 | 4.99 |
| 2.10                                | 2.05  | 2.01  | 1.96  | 1.93  | 1.91  | 1.89 | 1.87 | 1.86 | 1.83 | 1.80 |
| 2.60                                | 2.53  | 2.46  | 2.39  | 2.34  | 2.31  | 2.27 | 2.24 | 2.22 | 2.18 | 2.14 |
| 3.15                                | 3.05  | 2.95  | 2.84  | 2.78  | 2.73  | 2.67 | 2.64 | 2.61 | 2.55 | 2.50 |
| 3.94                                | 3.80  | 3.66  | 3.51  | 3.41  | 3.35  | 3.27 | 3.22 | 3.18 | 3.09 | 3.02 |
| 6.40                                | 6.13  | 5.85  | 5.56  | 5.38  | 5.25  | 5.10 | 5.00 | 4.94 | 4.77 | 4.62 |
| 2.06                                | 2.02  | 1.97  | 1.92  | 1.89  | 1.87  | 1.85 | 1.83 | 1.82 | 1.79 | 1.76 |
| 2.54                                | 2.48  | 2.40  | 2.33  | 2.28  | 2.25  | 2.20 | 2.18 | 2.16 | 2.11 | 2.07 |
| 3.06                                | 2.96  | 2.86  | 2.76  | 2.69  | 2.64  | 2.59 | 2.55 | 2.52 | 2.46 | 2.40 |
| 3.80                                | 3.67  | 3.52  | 3.37  | 3.28  | 3.21  | 3.13 | 3.08 | 3.05 | 2.96 | 2.88 |
| 6.08                                | 5.81  | 5.54  | 5.25  | 5.07  | 4.95  | 4.80 | 4.70 | 4.64 | 4.47 | 4.33 |
| 2.03                                | 1.99  | 1.94  | 1.89  | 1.86  | 1.84  | 1.81 | 1.79 | 1.78 | 1.75 | 1.72 |
| 2.49                                | 2.42  | 2.35  | 2.28  | 2.23  | 2.19  | 2.15 | 2.12 | 2.11 | 2.06 | 2.02 |
| 2.99                                | 2.89  | 2.79  | 2.68  | 2.61  | 2.57  | 2.51 | 2.47 | 2.45 | 2.38 | 2.32 |
| 3.69                                | 3.55  | 3.41  | 3.26  | 3.16  | 3.10  | 3.02 | 2.97 | 2.93 | 2.84 | 2.76 |
| 5.81                                | 5.55  | 5.27  | 4.99  | 4.82  | 4.70  | 4.54 | 4.45 | 4.39 | 4.23 | 4.08 |
| 2.00                                | 1.96  | 1.91  | 1.86  | 1.83  | 1.81  | 1.78 | 1.76 | 1.75 | 1.72 | 1.69 |
| 2.45                                | 2.38  | 2.31  | 2.23  | 2.18  | 2.15  | 2.10 | 2.08 | 2.06 | 2.01 | 1.97 |
| 2.92                                | 2.82  | 2.72  | 2.62  | 2.55  | 2.50  | 2.44 | 2.41 | 2.38 | 2.32 | 2.26 |
| 3.59                                | 3.46  | 3.31  | 3.16  | 3.07  | 3.00  | 2.92 | 2.87 | 2.83 | 2.75 | 2.66 |
| 5.58                                | 5.32  | 5.05  | 4.78  | 4.60  | 4.48  | 4.33 | 4.24 | 4.18 | 4.02 | 3.87 |

(Continued)

**TABLE E**

*F* critical values (continued)

|                                       |    | Degrees of freedom in the numerator |       |       |       |      |      |      |      |      |      |      |
|---------------------------------------|----|-------------------------------------|-------|-------|-------|------|------|------|------|------|------|------|
|                                       |    | 1                                   | 2     | 3     | 4     | 5    | 6    | 7    | 8    | 9    |      |      |
| <i>p</i>                              |    |                                     |       |       |       |      |      |      |      |      |      |      |
| Degrees of freedom in the denominator | 18 | .100                                | 3.01  | 2.62  | 2.42  | 2.29 | 2.20 | 2.13 | 2.08 | 2.04 | 2.00 |      |
|                                       |    | .050                                | 4.41  | 3.55  | 3.16  | 2.93 | 2.77 | 2.66 | 2.58 | 2.51 | 2.46 |      |
|                                       |    | .025                                | 5.98  | 4.56  | 3.95  | 3.61 | 3.38 | 3.22 | 3.10 | 3.01 | 2.93 |      |
|                                       |    | .010                                | 8.29  | 6.01  | 5.09  | 4.58 | 4.25 | 4.01 | 3.84 | 3.71 | 3.60 |      |
|                                       |    | .001                                | 15.38 | 10.39 | 8.49  | 7.46 | 6.81 | 6.35 | 6.02 | 5.76 | 5.56 |      |
|                                       |    | 19                                  | .100  | 2.99  | 2.61  | 2.40 | 2.27 | 2.18 | 2.11 | 2.06 | 2.02 | 1.98 |
|                                       |    |                                     | .050  | 4.38  | 3.52  | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.42 |
|                                       |    |                                     | .025  | 5.92  | 4.51  | 3.90 | 3.56 | 3.33 | 3.17 | 3.05 | 2.96 | 2.88 |
|                                       |    |                                     | .010  | 8.18  | 5.93  | 5.01 | 4.50 | 4.17 | 3.94 | 3.77 | 3.63 | 3.52 |
|                                       |    |                                     | .001  | 15.08 | 10.16 | 8.28 | 7.27 | 6.62 | 6.18 | 5.85 | 5.59 | 5.39 |
|                                       |    | 20                                  | .100  | 2.97  | 2.59  | 2.38 | 2.25 | 2.16 | 2.09 | 2.04 | 2.00 | 1.96 |
|                                       |    |                                     | .050  | 4.35  | 3.49  | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 |
|                                       |    |                                     | .025  | 5.87  | 4.46  | 3.86 | 3.51 | 3.29 | 3.13 | 3.01 | 2.91 | 2.84 |
|                                       |    |                                     | .010  | 8.10  | 5.85  | 4.94 | 4.43 | 4.10 | 3.87 | 3.70 | 3.56 | 3.46 |
|                                       |    |                                     | .001  | 14.82 | 9.95  | 8.10 | 7.10 | 6.46 | 6.02 | 5.69 | 5.44 | 5.24 |
|                                       |    | 21                                  | .100  | 2.96  | 2.57  | 2.36 | 2.23 | 2.14 | 2.08 | 2.02 | 1.98 | 1.95 |
|                                       |    |                                     | .050  | 4.32  | 3.47  | 3.07 | 2.84 | 2.68 | 2.57 | 2.49 | 2.42 | 2.37 |
|                                       |    |                                     | .025  | 5.83  | 4.42  | 3.82 | 3.48 | 3.25 | 3.09 | 2.97 | 2.87 | 2.80 |
|                                       |    |                                     | .010  | 8.02  | 5.78  | 4.87 | 4.37 | 4.04 | 3.81 | 3.64 | 3.51 | 3.40 |
|                                       |    |                                     | .001  | 14.59 | 9.77  | 7.94 | 6.95 | 6.32 | 5.88 | 5.56 | 5.31 | 5.11 |
|                                       |    | 22                                  | .100  | 2.95  | 2.56  | 2.35 | 2.22 | 2.13 | 2.06 | 2.01 | 1.97 | 1.93 |
|                                       |    |                                     | .050  | 4.30  | 3.44  | 3.05 | 2.82 | 2.66 | 2.55 | 2.46 | 2.40 | 2.34 |
|                                       |    |                                     | .025  | 5.79  | 4.38  | 3.78 | 3.44 | 3.22 | 3.05 | 2.93 | 2.84 | 2.76 |
|                                       |    |                                     | .010  | 7.95  | 5.72  | 4.82 | 4.31 | 3.99 | 3.76 | 3.59 | 3.45 | 3.35 |
|                                       |    |                                     | .001  | 14.38 | 9.61  | 7.80 | 6.81 | 6.19 | 5.76 | 5.44 | 5.19 | 4.99 |
|                                       |    | 23                                  | .100  | 2.94  | 2.55  | 2.34 | 2.21 | 2.11 | 2.05 | 1.99 | 1.95 | 1.92 |
|                                       |    |                                     | .050  | 4.28  | 3.42  | 3.03 | 2.80 | 2.64 | 2.53 | 2.44 | 2.37 | 2.32 |
|                                       |    | .025                                | 5.75  | 4.35  | 3.75  | 3.41 | 3.18 | 3.02 | 2.90 | 2.81 | 2.73 |      |
|                                       |    | .010                                | 7.88  | 5.66  | 4.76  | 4.26 | 3.94 | 3.71 | 3.54 | 3.41 | 3.30 |      |
|                                       |    | .001                                | 14.20 | 9.47  | 7.67  | 6.70 | 6.08 | 5.65 | 5.33 | 5.09 | 4.89 |      |
|                                       | 24 | .100                                | 2.93  | 2.54  | 2.33  | 2.19 | 2.10 | 2.04 | 1.98 | 1.94 | 1.91 |      |
|                                       |    | .050                                | 4.26  | 3.40  | 3.01  | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.30 |      |
|                                       |    | .025                                | 5.72  | 4.32  | 3.72  | 3.38 | 3.15 | 2.99 | 2.87 | 2.78 | 2.70 |      |
|                                       |    | .010                                | 7.82  | 5.61  | 4.72  | 4.22 | 3.90 | 3.67 | 3.50 | 3.36 | 3.26 |      |
|                                       |    | .001                                | 14.03 | 9.34  | 7.55  | 6.59 | 5.98 | 5.55 | 5.23 | 4.99 | 4.80 |      |
|                                       | 25 | .100                                | 2.92  | 2.53  | 2.32  | 2.18 | 2.09 | 2.02 | 1.97 | 1.93 | 1.89 |      |
|                                       |    | .050                                | 4.24  | 3.39  | 2.99  | 2.76 | 2.60 | 2.49 | 2.40 | 2.34 | 2.28 |      |
|                                       |    | .025                                | 5.69  | 4.29  | 3.69  | 3.35 | 3.13 | 2.97 | 2.85 | 2.75 | 2.68 |      |
|                                       |    | .010                                | 7.77  | 5.57  | 4.68  | 4.18 | 3.85 | 3.63 | 3.46 | 3.32 | 3.22 |      |
|                                       |    | .001                                | 13.88 | 9.22  | 7.45  | 6.49 | 5.89 | 5.46 | 5.15 | 4.91 | 4.71 |      |
|                                       | 26 | .100                                | 2.91  | 2.52  | 2.31  | 2.17 | 2.08 | 2.01 | 1.96 | 1.92 | 1.88 |      |
|                                       |    | .050                                | 4.23  | 3.37  | 2.98  | 2.74 | 2.59 | 2.47 | 2.39 | 2.32 | 2.27 |      |
|                                       |    | .025                                | 5.66  | 4.27  | 3.67  | 3.33 | 3.10 | 2.94 | 2.82 | 2.73 | 2.65 |      |
|                                       |    | .010                                | 7.72  | 5.53  | 4.64  | 4.14 | 3.82 | 3.59 | 3.42 | 3.29 | 3.18 |      |
|                                       |    | .001                                | 13.74 | 9.12  | 7.36  | 6.41 | 5.80 | 5.38 | 5.07 | 4.83 | 4.64 |      |
|                                       | 27 | .100                                | 2.90  | 2.51  | 2.30  | 2.17 | 2.07 | 2.00 | 1.95 | 1.91 | 1.87 |      |
|                                       |    | .050                                | 4.21  | 3.35  | 2.96  | 2.73 | 2.57 | 2.46 | 2.37 | 2.31 | 2.25 |      |
|                                       |    | .025                                | 5.63  | 4.24  | 3.65  | 3.31 | 3.08 | 2.92 | 2.80 | 2.71 | 2.63 |      |
|                                       |    | .010                                | 7.68  | 5.49  | 4.60  | 4.11 | 3.78 | 3.56 | 3.39 | 3.26 | 3.15 |      |
|                                       |    | .001                                | 13.61 | 9.02  | 7.27  | 6.33 | 5.73 | 5.31 | 5.00 | 4.76 | 4.57 |      |

**TABLE E****F critical values (continued)**

| Degrees of freedom in the numerator |      |      |      |      |      |      |      |      |      |      |
|-------------------------------------|------|------|------|------|------|------|------|------|------|------|
| 10                                  | 12   | 15   | 20   | 25   | 30   | 40   | 50   | 60   | 120  | 1000 |
| 1.98                                | 1.93 | 1.89 | 1.84 | 1.80 | 1.78 | 1.75 | 1.74 | 1.72 | 1.69 | 1.66 |
| 2.41                                | 2.34 | 2.27 | 2.19 | 2.14 | 2.11 | 2.06 | 2.04 | 2.02 | 1.97 | 1.92 |
| 2.87                                | 2.77 | 2.67 | 2.56 | 2.49 | 2.44 | 2.38 | 2.35 | 2.32 | 2.26 | 2.20 |
| 3.51                                | 3.37 | 3.23 | 3.08 | 2.98 | 2.92 | 2.84 | 2.78 | 2.75 | 2.66 | 2.58 |
| 5.39                                | 5.13 | 4.87 | 4.59 | 4.42 | 4.30 | 4.15 | 4.06 | 4.00 | 3.84 | 3.69 |
| 1.96                                | 1.91 | 1.86 | 1.81 | 1.78 | 1.76 | 1.73 | 1.71 | 1.70 | 1.67 | 1.64 |
| 2.38                                | 2.31 | 2.23 | 2.16 | 2.11 | 2.07 | 2.03 | 2.00 | 1.98 | 1.93 | 1.88 |
| 2.82                                | 2.72 | 2.62 | 2.51 | 2.44 | 2.39 | 2.33 | 2.30 | 2.27 | 2.20 | 2.14 |
| 3.43                                | 3.30 | 3.15 | 3.00 | 2.91 | 2.84 | 2.76 | 2.71 | 2.67 | 2.58 | 2.50 |
| 5.22                                | 4.97 | 4.70 | 4.43 | 4.26 | 4.14 | 3.99 | 3.90 | 3.84 | 3.68 | 3.53 |
| 1.94                                | 1.89 | 1.84 | 1.79 | 1.76 | 1.74 | 1.71 | 1.69 | 1.68 | 1.64 | 1.61 |
| 2.35                                | 2.28 | 2.20 | 2.12 | 2.07 | 2.04 | 1.99 | 1.97 | 1.95 | 1.90 | 1.85 |
| 2.77                                | 2.68 | 2.57 | 2.46 | 2.40 | 2.35 | 2.29 | 2.25 | 2.22 | 2.16 | 2.09 |
| 3.37                                | 3.23 | 3.09 | 2.94 | 2.84 | 2.78 | 2.69 | 2.64 | 2.61 | 2.52 | 2.43 |
| 5.08                                | 4.82 | 4.56 | 4.29 | 4.12 | 4.00 | 3.86 | 3.77 | 3.70 | 3.54 | 3.40 |
| 1.92                                | 1.87 | 1.83 | 1.78 | 1.74 | 1.72 | 1.69 | 1.67 | 1.66 | 1.62 | 1.59 |
| 2.32                                | 2.25 | 2.18 | 2.10 | 2.05 | 2.01 | 1.96 | 1.94 | 1.92 | 1.87 | 1.82 |
| 2.73                                | 2.64 | 2.53 | 2.42 | 2.36 | 2.31 | 2.25 | 2.21 | 2.18 | 2.11 | 2.05 |
| 3.31                                | 3.17 | 3.03 | 2.88 | 2.79 | 2.72 | 2.64 | 2.58 | 2.55 | 2.46 | 2.37 |
| 4.95                                | 4.70 | 4.44 | 4.17 | 4.00 | 3.88 | 3.74 | 3.64 | 3.58 | 3.42 | 3.28 |
| 1.90                                | 1.86 | 1.81 | 1.76 | 1.73 | 1.70 | 1.67 | 1.65 | 1.64 | 1.60 | 1.57 |
| 2.30                                | 2.23 | 2.15 | 2.07 | 2.02 | 1.98 | 1.94 | 1.91 | 1.89 | 1.84 | 1.79 |
| 2.70                                | 2.60 | 2.50 | 2.39 | 2.32 | 2.27 | 2.21 | 2.17 | 2.14 | 2.08 | 2.01 |
| 3.26                                | 3.12 | 2.98 | 2.83 | 2.73 | 2.67 | 2.58 | 2.53 | 2.50 | 2.40 | 2.32 |
| 4.83                                | 4.58 | 4.33 | 4.06 | 3.89 | 3.78 | 3.63 | 3.54 | 3.48 | 3.32 | 3.17 |
| 1.89                                | 1.84 | 1.80 | 1.74 | 1.71 | 1.69 | 1.66 | 1.64 | 1.62 | 1.59 | 1.55 |
| 2.27                                | 2.20 | 2.13 | 2.05 | 2.00 | 1.96 | 1.91 | 1.88 | 1.86 | 1.81 | 1.76 |
| 2.67                                | 2.57 | 2.47 | 2.36 | 2.29 | 2.24 | 2.18 | 2.14 | 2.11 | 2.04 | 1.98 |
| 3.21                                | 3.07 | 2.93 | 2.78 | 2.69 | 2.62 | 2.54 | 2.48 | 2.45 | 2.35 | 2.27 |
| 4.73                                | 4.48 | 4.23 | 3.96 | 3.79 | 3.68 | 3.53 | 3.44 | 3.38 | 3.22 | 3.08 |
| 1.88                                | 1.83 | 1.78 | 1.73 | 1.70 | 1.67 | 1.64 | 1.62 | 1.61 | 1.57 | 1.54 |
| 2.25                                | 2.18 | 2.11 | 2.03 | 1.97 | 1.94 | 1.89 | 1.86 | 1.84 | 1.79 | 1.74 |
| 2.64                                | 2.54 | 2.44 | 2.33 | 2.26 | 2.21 | 2.15 | 2.11 | 2.08 | 2.01 | 1.94 |
| 3.17                                | 3.03 | 2.89 | 2.74 | 2.64 | 2.58 | 2.49 | 2.44 | 2.40 | 2.31 | 2.22 |
| 4.64                                | 4.39 | 4.14 | 3.87 | 3.71 | 3.59 | 3.45 | 3.36 | 3.29 | 3.14 | 2.99 |
| 1.87                                | 1.82 | 1.77 | 1.72 | 1.68 | 1.66 | 1.63 | 1.61 | 1.59 | 1.56 | 1.52 |
| 2.24                                | 2.16 | 2.09 | 2.01 | 1.96 | 1.92 | 1.87 | 1.84 | 1.82 | 1.77 | 1.72 |
| 2.61                                | 2.51 | 2.41 | 2.30 | 2.23 | 2.18 | 2.12 | 2.08 | 2.05 | 1.98 | 1.91 |
| 3.13                                | 2.99 | 2.85 | 2.70 | 2.60 | 2.54 | 2.45 | 2.40 | 2.36 | 2.27 | 2.18 |
| 4.56                                | 4.31 | 4.06 | 3.79 | 3.63 | 3.52 | 3.37 | 3.28 | 3.22 | 3.06 | 2.91 |
| 1.86                                | 1.81 | 1.76 | 1.71 | 1.67 | 1.65 | 1.61 | 1.59 | 1.58 | 1.54 | 1.51 |
| 2.22                                | 2.15 | 2.07 | 1.99 | 1.94 | 1.90 | 1.85 | 1.82 | 1.80 | 1.75 | 1.70 |
| 2.59                                | 2.49 | 2.39 | 2.28 | 2.21 | 2.16 | 2.09 | 2.05 | 2.03 | 1.95 | 1.89 |
| 3.09                                | 2.96 | 2.81 | 2.66 | 2.57 | 2.50 | 2.42 | 2.36 | 2.33 | 2.23 | 2.14 |
| 4.48                                | 4.24 | 3.99 | 3.72 | 3.56 | 3.44 | 3.30 | 3.21 | 3.15 | 2.99 | 2.84 |
| 1.85                                | 1.80 | 1.75 | 1.70 | 1.66 | 1.64 | 1.60 | 1.58 | 1.57 | 1.53 | 1.50 |
| 2.20                                | 2.13 | 2.06 | 1.97 | 1.92 | 1.88 | 1.84 | 1.81 | 1.79 | 1.73 | 1.68 |
| 2.57                                | 2.47 | 2.36 | 2.25 | 2.18 | 2.13 | 2.07 | 2.03 | 2.00 | 1.93 | 1.86 |
| 3.06                                | 2.93 | 2.78 | 2.63 | 2.54 | 2.47 | 2.38 | 2.33 | 2.29 | 2.20 | 2.11 |
| 4.41                                | 4.17 | 3.92 | 3.66 | 3.49 | 3.38 | 3.23 | 3.14 | 3.08 | 2.92 | 2.78 |

(Continued)

**TABLE E**

*F* critical values (continued)

|                                       |      | Degrees of freedom in the numerator |       |      |      |      |      |      |      |      |      |
|---------------------------------------|------|-------------------------------------|-------|------|------|------|------|------|------|------|------|
| <i>p</i>                              |      | 1                                   | 2     | 3    | 4    | 5    | 6    | 7    | 8    | 9    |      |
| Degrees of freedom in the denominator | 28   | .100                                | 2.89  | 2.50 | 2.29 | 2.16 | 2.06 | 2.00 | 1.94 | 1.90 | 1.87 |
|                                       |      | .050                                | 4.20  | 3.34 | 2.95 | 2.71 | 2.56 | 2.45 | 2.36 | 2.29 | 2.24 |
|                                       |      | .025                                | 5.61  | 4.22 | 3.63 | 3.29 | 3.06 | 2.90 | 2.78 | 2.69 | 2.61 |
|                                       |      | .010                                | 7.64  | 5.45 | 4.57 | 4.07 | 3.75 | 3.53 | 3.36 | 3.23 | 3.12 |
|                                       |      | .001                                | 13.50 | 8.93 | 7.19 | 6.25 | 5.66 | 5.24 | 4.93 | 4.69 | 4.50 |
|                                       | 29   | .100                                | 2.89  | 2.50 | 2.28 | 2.15 | 2.06 | 1.99 | 1.93 | 1.89 | 1.86 |
|                                       |      | .050                                | 4.18  | 3.33 | 2.93 | 2.70 | 2.55 | 2.43 | 2.35 | 2.28 | 2.22 |
|                                       |      | .025                                | 5.59  | 4.20 | 3.61 | 3.27 | 3.04 | 2.88 | 2.76 | 2.67 | 2.59 |
|                                       |      | .010                                | 7.60  | 5.42 | 4.54 | 4.04 | 3.73 | 3.50 | 3.33 | 3.20 | 3.09 |
|                                       |      | .001                                | 13.39 | 8.85 | 7.12 | 6.19 | 5.59 | 5.18 | 4.87 | 4.64 | 4.45 |
|                                       | 30   | .100                                | 2.88  | 2.49 | 2.28 | 2.14 | 2.05 | 1.98 | 1.93 | 1.88 | 1.85 |
|                                       |      | .050                                | 4.17  | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.33 | 2.27 | 2.21 |
|                                       |      | .025                                | 5.57  | 4.18 | 3.59 | 3.25 | 3.03 | 2.87 | 2.75 | 2.65 | 2.57 |
|                                       |      | .010                                | 7.56  | 5.39 | 4.51 | 4.02 | 3.70 | 3.47 | 3.30 | 3.17 | 3.07 |
|                                       |      | .001                                | 13.29 | 8.77 | 7.05 | 6.12 | 5.53 | 5.12 | 4.82 | 4.58 | 4.39 |
|                                       | 40   | .100                                | 2.84  | 2.44 | 2.23 | 2.09 | 2.00 | 1.93 | 1.87 | 1.83 | 1.79 |
|                                       |      | .050                                | 4.08  | 3.23 | 2.84 | 2.61 | 2.45 | 2.34 | 2.25 | 2.18 | 2.12 |
|                                       |      | .025                                | 5.42  | 4.05 | 3.46 | 3.13 | 2.90 | 2.74 | 2.62 | 2.53 | 2.45 |
|                                       |      | .010                                | 7.31  | 5.18 | 4.31 | 3.83 | 3.51 | 3.29 | 3.12 | 2.99 | 2.89 |
|                                       |      | .001                                | 12.61 | 8.25 | 6.59 | 5.70 | 5.13 | 4.73 | 4.44 | 4.21 | 4.02 |
| 50                                    | .100 | 2.81                                | 2.41  | 2.20 | 2.06 | 1.97 | 1.90 | 1.84 | 1.80 | 1.76 |      |
|                                       | .050 | 4.03                                | 3.18  | 2.79 | 2.56 | 2.40 | 2.29 | 2.20 | 2.13 | 2.07 |      |
|                                       | .025 | 5.34                                | 3.97  | 3.39 | 3.05 | 2.83 | 2.67 | 2.55 | 2.46 | 2.38 |      |
|                                       | .010 | 7.17                                | 5.06  | 4.20 | 3.72 | 3.41 | 3.19 | 3.02 | 2.89 | 2.78 |      |
|                                       | .001 | 12.22                               | 7.96  | 6.34 | 5.46 | 4.90 | 4.51 | 4.22 | 4.00 | 3.82 |      |
| 60                                    | .100 | 2.79                                | 2.39  | 2.18 | 2.04 | 1.95 | 1.87 | 1.82 | 1.77 | 1.74 |      |
|                                       | .050 | 4.00                                | 3.15  | 2.76 | 2.53 | 2.37 | 2.25 | 2.17 | 2.10 | 2.04 |      |
|                                       | .025 | 5.29                                | 3.93  | 3.34 | 3.01 | 2.79 | 2.63 | 2.51 | 2.41 | 2.33 |      |
|                                       | .010 | 7.08                                | 4.98  | 4.13 | 3.65 | 3.34 | 3.12 | 2.95 | 2.82 | 2.72 |      |
|                                       | .001 | 11.97                               | 7.77  | 6.17 | 5.31 | 4.76 | 4.37 | 4.09 | 3.86 | 3.69 |      |
| 100                                   | .100 | 2.76                                | 2.36  | 2.14 | 2.00 | 1.91 | 1.83 | 1.78 | 1.73 | 1.69 |      |
|                                       | .050 | 3.94                                | 3.09  | 2.70 | 2.46 | 2.31 | 2.19 | 2.10 | 2.03 | 1.97 |      |
|                                       | .025 | 5.18                                | 3.83  | 3.25 | 2.92 | 2.70 | 2.54 | 2.42 | 2.32 | 2.24 |      |
|                                       | .010 | 6.90                                | 4.82  | 3.98 | 3.51 | 3.21 | 2.99 | 2.82 | 2.69 | 2.59 |      |
|                                       | .001 | 11.50                               | 7.41  | 5.86 | 5.02 | 4.48 | 4.11 | 3.83 | 3.61 | 3.44 |      |
| 200                                   | .100 | 2.73                                | 2.33  | 2.11 | 1.97 | 1.88 | 1.80 | 1.75 | 1.70 | 1.66 |      |
|                                       | .050 | 3.89                                | 3.04  | 2.65 | 2.42 | 2.26 | 2.14 | 2.06 | 1.98 | 1.93 |      |
|                                       | .025 | 5.10                                | 3.76  | 3.18 | 2.85 | 2.63 | 2.47 | 2.35 | 2.26 | 2.18 |      |
|                                       | .010 | 6.76                                | 4.71  | 3.88 | 3.41 | 3.11 | 2.89 | 2.73 | 2.60 | 2.50 |      |
|                                       | .001 | 11.15                               | 7.15  | 5.63 | 4.81 | 4.29 | 3.92 | 3.65 | 3.43 | 3.26 |      |
| 1000                                  | .100 | 2.71                                | 2.31  | 2.09 | 1.95 | 1.85 | 1.78 | 1.72 | 1.68 | 1.64 |      |
|                                       | .050 | 3.85                                | 3.00  | 2.61 | 2.38 | 2.22 | 2.11 | 2.02 | 1.95 | 1.89 |      |
|                                       | .025 | 5.04                                | 3.70  | 3.13 | 2.80 | 2.58 | 2.42 | 2.30 | 2.20 | 2.13 |      |
|                                       | .010 | 6.66                                | 4.63  | 3.80 | 3.34 | 3.04 | 2.82 | 2.66 | 2.53 | 2.43 |      |
|                                       | .001 | 10.89                               | 6.96  | 5.46 | 4.65 | 4.14 | 3.78 | 3.51 | 3.30 | 3.13 |      |

**TABLE E****F critical values (continued)**

| Degrees of freedom in the numerator |      |      |      |      |      |      |      |      |      |      |
|-------------------------------------|------|------|------|------|------|------|------|------|------|------|
| 10                                  | 12   | 15   | 20   | 25   | 30   | 40   | 50   | 60   | 120  | 1000 |
| 1.84                                | 1.79 | 1.74 | 1.69 | 1.65 | 1.63 | 1.59 | 1.57 | 1.56 | 1.52 | 1.48 |
| 2.19                                | 2.12 | 2.04 | 1.96 | 1.91 | 1.87 | 1.82 | 1.79 | 1.77 | 1.71 | 1.66 |
| 2.55                                | 2.45 | 2.34 | 2.23 | 2.16 | 2.11 | 2.05 | 2.01 | 1.98 | 1.91 | 1.84 |
| 3.03                                | 2.90 | 2.75 | 2.60 | 2.51 | 2.44 | 2.35 | 2.30 | 2.26 | 2.17 | 2.08 |
| 4.35                                | 4.11 | 3.86 | 3.60 | 3.43 | 3.32 | 3.18 | 3.09 | 3.02 | 2.86 | 2.72 |
| 1.83                                | 1.78 | 1.73 | 1.68 | 1.64 | 1.62 | 1.58 | 1.56 | 1.55 | 1.51 | 1.47 |
| 2.18                                | 2.10 | 2.03 | 1.94 | 1.89 | 1.85 | 1.81 | 1.77 | 1.75 | 1.70 | 1.65 |
| 2.53                                | 2.43 | 2.32 | 2.21 | 2.14 | 2.09 | 2.03 | 1.99 | 1.96 | 1.89 | 1.82 |
| 3.00                                | 2.87 | 2.73 | 2.57 | 2.48 | 2.41 | 2.33 | 2.27 | 2.23 | 2.14 | 2.05 |
| 4.29                                | 4.05 | 3.80 | 3.54 | 3.38 | 3.27 | 3.12 | 3.03 | 2.97 | 2.81 | 2.66 |
| 1.82                                | 1.77 | 1.72 | 1.67 | 1.63 | 1.61 | 1.57 | 1.55 | 1.54 | 1.50 | 1.46 |
| 2.16                                | 2.09 | 2.01 | 1.93 | 1.88 | 1.84 | 1.79 | 1.76 | 1.74 | 1.68 | 1.63 |
| 2.51                                | 2.41 | 2.31 | 2.20 | 2.12 | 2.07 | 2.01 | 1.97 | 1.94 | 1.87 | 1.80 |
| 2.98                                | 2.84 | 2.70 | 2.55 | 2.45 | 2.39 | 2.30 | 2.25 | 2.21 | 2.11 | 2.02 |
| 4.24                                | 4.00 | 3.75 | 3.49 | 3.33 | 3.22 | 3.07 | 2.98 | 2.92 | 2.76 | 2.61 |
| 1.76                                | 1.71 | 1.66 | 1.61 | 1.57 | 1.54 | 1.51 | 1.48 | 1.47 | 1.42 | 1.38 |
| 2.08                                | 2.00 | 1.92 | 1.84 | 1.78 | 1.74 | 1.69 | 1.66 | 1.64 | 1.58 | 1.52 |
| 2.39                                | 2.29 | 2.18 | 2.07 | 1.99 | 1.94 | 1.88 | 1.83 | 1.80 | 1.72 | 1.65 |
| 2.80                                | 2.66 | 2.52 | 2.37 | 2.27 | 2.20 | 2.11 | 2.06 | 2.02 | 1.92 | 1.82 |
| 3.87                                | 3.64 | 3.40 | 3.14 | 2.98 | 2.87 | 2.73 | 2.64 | 2.57 | 2.41 | 2.25 |
| 1.73                                | 1.68 | 1.63 | 1.57 | 1.53 | 1.50 | 1.46 | 1.44 | 1.42 | 1.38 | 1.33 |
| 2.03                                | 1.95 | 1.87 | 1.78 | 1.73 | 1.69 | 1.63 | 1.60 | 1.58 | 1.51 | 1.45 |
| 2.32                                | 2.22 | 2.11 | 1.99 | 1.92 | 1.87 | 1.80 | 1.75 | 1.72 | 1.64 | 1.56 |
| 2.70                                | 2.56 | 2.42 | 2.27 | 2.17 | 2.10 | 2.01 | 1.95 | 1.91 | 1.80 | 1.70 |
| 3.67                                | 3.44 | 3.20 | 2.95 | 2.79 | 2.68 | 2.53 | 2.44 | 2.38 | 2.21 | 2.05 |
| 1.71                                | 1.66 | 1.60 | 1.54 | 1.50 | 1.48 | 1.44 | 1.41 | 1.40 | 1.35 | 1.30 |
| 1.99                                | 1.92 | 1.84 | 1.75 | 1.69 | 1.65 | 1.59 | 1.56 | 1.53 | 1.47 | 1.40 |
| 2.27                                | 2.17 | 2.06 | 1.94 | 1.87 | 1.82 | 1.74 | 1.70 | 1.67 | 1.58 | 1.49 |
| 2.63                                | 2.50 | 2.35 | 2.20 | 2.10 | 2.03 | 1.94 | 1.88 | 1.84 | 1.73 | 1.62 |
| 3.54                                | 3.32 | 3.08 | 2.83 | 2.67 | 2.55 | 2.41 | 2.32 | 2.25 | 2.08 | 1.92 |
| 1.66                                | 1.61 | 1.56 | 1.49 | 1.45 | 1.42 | 1.38 | 1.35 | 1.34 | 1.28 | 1.22 |
| 1.93                                | 1.85 | 1.77 | 1.68 | 1.62 | 1.57 | 1.52 | 1.48 | 1.45 | 1.38 | 1.30 |
| 2.18                                | 2.08 | 1.97 | 1.85 | 1.77 | 1.71 | 1.64 | 1.59 | 1.56 | 1.46 | 1.36 |
| 2.50                                | 2.37 | 2.22 | 2.07 | 1.97 | 1.89 | 1.80 | 1.74 | 1.69 | 1.57 | 1.45 |
| 3.30                                | 3.07 | 2.84 | 2.59 | 2.43 | 2.32 | 2.17 | 2.08 | 2.01 | 1.83 | 1.64 |
| 1.63                                | 1.58 | 1.52 | 1.46 | 1.41 | 1.38 | 1.34 | 1.31 | 1.29 | 1.23 | 1.16 |
| 1.88                                | 1.80 | 1.72 | 1.62 | 1.56 | 1.52 | 1.46 | 1.41 | 1.39 | 1.30 | 1.21 |
| 2.11                                | 2.01 | 1.90 | 1.78 | 1.70 | 1.64 | 1.56 | 1.51 | 1.47 | 1.37 | 1.25 |
| 2.41                                | 2.27 | 2.13 | 1.97 | 1.87 | 1.79 | 1.69 | 1.63 | 1.58 | 1.45 | 1.30 |
| 3.12                                | 2.90 | 2.67 | 2.42 | 2.26 | 2.15 | 2.00 | 1.90 | 1.83 | 1.64 | 1.43 |
| 1.61                                | 1.55 | 1.49 | 1.43 | 1.38 | 1.35 | 1.30 | 1.27 | 1.25 | 1.18 | 1.08 |
| 1.84                                | 1.76 | 1.68 | 1.58 | 1.52 | 1.47 | 1.41 | 1.36 | 1.33 | 1.24 | 1.11 |
| 2.06                                | 1.96 | 1.85 | 1.72 | 1.64 | 1.58 | 1.50 | 1.45 | 1.41 | 1.29 | 1.13 |
| 2.34                                | 2.20 | 2.06 | 1.90 | 1.79 | 1.72 | 1.61 | 1.54 | 1.50 | 1.35 | 1.16 |
| 2.99                                | 2.77 | 2.54 | 2.30 | 2.14 | 2.02 | 1.87 | 1.77 | 1.69 | 1.49 | 1.22 |