Name:
Enrollment No:

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, January 2021

Programme Name: B. Tech. (All SOE) **Semester : I**
Course Name : Mathematics I **Time : 03 hrs**
Course Code: MATH 1026 **Max. Marks : 100**

Section A (All questions are compulsory, each question is of 5 marks)		
1.	The Fourier cosine series for an even function f(x) is given by $$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(nx)$$ The value of the coefficient a_2 for the function $f(x) = cos^2(x)$ in $[0, \pi]$ is A. 0 B. 0.5 C. -0.5 D. 1	**CO4**
2.	For a matrix A of order 2×2, which is FALSE? A. $\det(A) = 0 \Longrightarrow Rank(A) \neq 2.$ B. $\det(A^{-1}) \neq 0 \Longrightarrow Rank(A) = 2.$ C. $\det(A^{-1}) = 1 \Longrightarrow Rank(A) = 1.$ D. $\det(A^{-1}) = 2 \Longrightarrow Rank(A) = 2.$	**CO1**
3.	Which of the following is not Dirichlet's condition for the Fourier series expansion of function $f(x)$? A. $f(x)$ is periodic, single valued, finite B. $f(x)$ has finite number of discontinuities C. $f(x)$ has finite number of maxima and minima D. $f(x)$ has infinite number of discontinuities	**CO4**
4.	If the vector function $\vec{F} = (3y - p_1 z)\hat{i} + (p_2 x - 2z)\hat{j} + (p_3 y + z)\hat{k}$ is irrotational, then the values of the constants p_1, p_2, p_3 respectively, are A. 0.3, -2.5, 0 B. 0, 3, 2 C. 0, 0.33, 0.5 D. 4, 3, 2	**CO3**
5.	The Fourier series of the function $f(x) = sin^2 x$ is A. $sin\, x + sin\, 2x$ B. $1 - cos\, 2x$ C. $sin\, 2x + cos\, 2x$ D. $0.5 - 0.5\, cos\, 2x$	**CO4**

6.	The function $f(x, y) = x^2y - 3xy + 2y + x$ has A. No local extremum B. One local maximum but no local minimum C. One local minimum but no local maximum D. One local maximum and one local minimum	**CO2**
SECTION B **(All questions are compulsory and Q11 has internal choices, each question is of 10 marks)**		
7.	Find the directional derivatives of $x^2 y^2 z^2$ at the point (1, 1, -1) in the direction of the tangent to the curve $x = e^t$, $y = \sin 2t + 1$, $z = 1 - \cos t$ at $t = 0$.	**CO3**
8.	Evaluate $\iint_R x\,dx\,dy$ over the region bounded by $y^2 = x$ and the lines $x + y = 2$, $x = 0$ and $x = 1$.	**CO2**
9.	A vector field is given by $\vec{F} = \sin y\,\hat{\imath} + x(1 + \cos y)\hat{\jmath}$. Evaluate the line integral over a circular path $x^2 + y^2 = a^2, z = 0$.	**CO3**
10.	If $u = \tan^{-1}\frac{x^3+y^3}{x-y}$, show that $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = (1 - 4sin^2u)\sin 2u$.	**CO2**
11.	Find the Fourier series representing $f(x) = x\sin x, 0 < x < 2\pi$ **OR** Given that $f(x) = x + x^2$ for $-\pi < x < \pi$, find the Fourier expression of f(x). Deduce that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots\ldots\ldots$	**CO4**
SECTION C **(Q12 is of 20 marks and it has internal choices)**		
12	Suppose $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is a matrix with real entries with $a_{12} \neq 0, a_{21} \neq 0$. Prove that a. If A has repeated eigenvalues then $\det(A)$ is non-negative. b. If a_{12} and a_{21} have same sign then A has real and distinct eigenvalues. Is the converse also true? Give suitable reason or a counterexample to support your answer. c. Take $a_{11} = a_{22} = 1$ and $a_{12} = a_{21} = \epsilon > 0$. If $\lambda_{\max}$ and $\lambda_{\min}$, respectively are the largest and smallest eigenvalues of A then find $\lim_{\epsilon \to 0^+} \frac{\lambda_{\max}}{\lambda_{\min}}$. **OR** Suppose $0 \le \alpha \le 2\pi, 0 \le \beta \le 2\pi, 0 \le \gamma \le 2\pi$. Find the number of solutions of the system $\sin\alpha + 2\cos\beta + 3\tan\gamma = 0$ $2\sin\alpha + 5\cos\beta + 3\tan\gamma = 0$ $-\sin\alpha - 5\cos\beta + 5\tan\gamma = 0$	**CO1**