Name:

Enrollment No:

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, January 2021

Programme Name: B.Sc. Mathematics (Hons.) **Semester : I**
Course Name : Algebra **Time : 03 hrs**
Course Code: MATH 1032 **Max. Marks : 100**

Section A (All questions are compulsory)			
1.	If $1, \omega, \omega^2$ are the cube roots of unity, then the roots of $(x-1)^3 + 8 = 0$ are A. $-1, 1+2\omega, 1-2\omega^2$ B. $-1, 1+2\omega, 1+2\omega^2$ C. $-1, 1-2\omega, 1+2\omega^2$ D. $-1, 1-2\omega, 1-2\omega^2$	**[5]**	**CO1**
2.	The function $f(x) = \frac{x}{x^2+1}$ from $\mathcal{R}$ to $\mathcal{R}$, where $\mathcal{R}$ is the set of real numbers, is A. one-one and onto B. one-one but not onto C. not one-one but onto D. neither one-one nor onto	**[5]**	**CO2**
3.	Consider the equations of two planes $x+y+z=3$ and $2x+3y+z=6$. Choose the correct option. A. The planes have a unique point of intersection. B. The planes have a line of intersection. C. The planes do not intersect. D. The plane $2x+3y+z=6$ passes through the origin.	**[5]**	**CO3**
4.	The rank of the matrix $A = \begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix}$ A. 1 B. 2 C. 3 D. 4	**[5]**	**CO3**

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5.	If A is a square matrix of order 2 having only integer entries. Then, which of the following option cannot be the characteristic polynomial of A? A. $\lambda^2 - 2\lambda$ B. $2\lambda^2 - \lambda$ C. $\lambda^2 - 2\lambda + 1$ D. λ^2	**[5]**	**CO4**
6.	Which of the following mapping $T: \mathcal{R}^2 \rightarrow \mathcal{R}^2$, where $\mathcal{R}$ is the set of real numbers, is not a linear transformation. A. $T(x, y) = (y, x)$ B. $T(x, y) = (x + y, x)$ C. $T(x, y) = (x + 1, y)$ D. $T(x, y) = (0,0)$	**[5]**	**CO4**
SECTION B **(Q1-Q5 are compulsory and Q5 has internal choices)**			
1.	Compute $z^n + \frac{1}{z^n}$, if $z + \frac{1}{z} = \sqrt{3}$	**[10]**	**CO1**
2.	Determine all solutions in the positive integers of the following Diophantine equation: $$18x + 5y = 48$$	**[10]**	**CO2**
3.	If $A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$ always satisfies the matrix equation $A^3 - A^2 + A = kI$, then find the value of the constant k. Use the relation $A^3 - A^2 + A = kI$ to find A^5.	**[10]**	**CO4**
4.	Let $T: \mathcal{R}^2 \rightarrow \mathcal{R}^3$, where $\mathcal{R}$ is the set of real numbers, is a linear transformation, defines as $$T(x, y) = (3x + y, 5x + 7y, x + 3y)$$ Find the matrix of the linear transformation and use the matrix to show that the transformation is one-one.	**[10]**	**CO4**

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5.	Determine if the following homogeneous system has a nontrivial (nonzero) solution. Then describe the solution set. $$3x_1 + 5x_2 - 4x_3 = 0$$ $$-3x_1 - 2x_2 + 4x_3 = 0$$ $$6x_1 + x_2 - 8x_3 = 0$$ **OR** Consider three vectors $v_1 = \begin{bmatrix} 3 \\ 0 \\ -6 \end{bmatrix}$, $v_2 = \begin{bmatrix} -4 \\ 1 \\ 7 \end{bmatrix}$ and $v_3 = \begin{bmatrix} -2 \\ 1 \\ 5 \end{bmatrix}$. Determine, if v_1, v_2 and $v_3$are linearly independent.	**[10]**	**CO3**
SECTION C **(Q1 is compulsory and has an internal choices)**			
1	a. Find the inverse of following matrix using elementary row operations $$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{bmatrix}$$ b. Let $A = \begin{bmatrix} 1 & -3 & -4 \\ -4 & 6 & -2 \\ -3 & 7 & 6 \end{bmatrix}$ and $b = \begin{bmatrix} 3 \\ 3 \\ -4 \end{bmatrix}$. Determine whether b is in the column space of A. **OR** Let T be a function $T : R^3 \rightarrow R^3$ by $T(x, y, z) = (x - y + 2z, 2x + y, -x - 2y + 2z)$. I. Verify that T is a linear transformation II. If (a, b, c) is a vector in R^3, what are the conditions on a, b and c that the vector be in the range of T? III. What are the conditions on a, b and c that the vector be in the null space of T?	**[20]**	**CO4**