

Name:

Enrolment No:



**UNIVERSITY OF PETROLEUM AND ENERGY STUDIES**

**End Examinations (Online Mode), Jan-Feb 2021**

**Course:** Mathematics

**Semester:** I

**Program:** BCA-Spl. BFSI

**Time:** 3 Hrs

**Course Code:** MATH1037

**Max. Marks:** 100

**SECTION - A**

**6 x 5 = 30 Marks**

**1. Each Question will carry 5 Marks**

**2. Instruction: Select the correct option(s)**

Q 1	The real roots of the quadratic equation $x^4 - 8x^2 + 15 = 0$ are A. $-\sqrt{3}, +\sqrt{3}, -\sqrt{5}, +\sqrt{5}$ B. $-\sqrt{2}, +\sqrt{2}, -\sqrt{6}, +\sqrt{6}$ C. $+2\sqrt{2}, -2\sqrt{2}, -6, +6$ D. None of these	CO1
Q 2	If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ , then the value of $A^2 - 5A + 7I$ is A. $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ B. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ C. $\begin{bmatrix} 2 & -2 \\ 3 & -4 \end{bmatrix}$ D. None of these	CO2
Q 3	For what value of $k$ , the rank of the matrix $\begin{bmatrix} 2 & 1 & -1 \\ 1 & 4 & 2 \\ 3 & 5 & k \end{bmatrix}$ is 2? A. $-1$ B. $0$ C. $1$ D. None of these	CO2
Q 4	The value of $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}$ is A. $1$ B. $2$ C. $3$ D. None of these	CO3
Q 5	If $y = u^2 + 1$ and $u = x^3 + x + 4$ , then the value of $\frac{dy}{dx}$ is A. $6x^5 + 6x^3 + 4x^2 + 2x + 8$ B. $6x^5 + 8x^3 + 24x^2 + 2x + 8$ C. $6x^5 + 8x^3 + 4x^2 + 2x + 8$ D. None of these	CO3
Q 6	A bag contains 8 white and 6 red balls. The probability of drawing two balls of the same color is A. $41/91$ B. $42/91$ C. $43/91$ D. None of these	CO4

<b>SECTION – B</b>		<b>10 x 5 = 50 Marks</b>
<p><b>1. Each question will carry 10 marks</b></p> <p><b>2. Instruction: Answer on a separate white sheet, upload the solution as image.</b></p>		
Q 7	<p>(a) Solve the equation <math>x^2 - 9x + 14 = 0</math> by completing the square method.</p> <p>(b) A total of Rs. 3,300 is raised by collecting equal amounts from a certain number of people. If there were 22 people more, each person would have to contribute Rs. 200 less to raise the same amount. How many people actually contributed?</p>	CO1
Q 8	<p>If <math>A = \begin{bmatrix} 2 &amp; 1 &amp; 3 \\ 3 &amp; 1 &amp; 2 \\ 1 &amp; 2 &amp; 3 \end{bmatrix}</math>, verify that <math>A(adj A) = (adj A)A =  A I_3</math>.</p>	CO2
Q 9	<p>Define the Rank of a matrix. Reduce the following matrix into its Echelon form and hence find its rank.</p> $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$	CO2
Q10	<p>Prove that <math>\int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2+b^2} [a \cos bx + b \sin bx] + C</math>.</p>	CO3
Q 11	<p>There are three bags: first containing 1 white, 2 red, 3 green balls; second containing 2 white, 3 red, 1 green balls; third containing 3 white, 1 red, 2 green balls. Two balls are drawn from a bag chosen at random. These are found to be 1 white and 1 red. Find the probability that the balls so drawn came from the second bag.</p>	CO4
<b>Section – C</b>		<b>1 x 20 = 20 Marks</b>
<p><b>1. Each Question carries 20 Marks.</b></p> <p><b>2. Instruction: Answer on a separate white sheet, upload the solution as image.</b></p>		
Q 12	<p><b>A:</b> Investigate the values of <math>\lambda</math> and <math>\mu</math> so that the equations <math>2x + 3y + 5z = 9</math>, <math>7x + 3y - 2z = 8</math>, and <math>2x + 3y + \lambda z = \mu</math> have (i) no solution (ii) a unique solution and (iii) an infinite number of solutions.</p> <p><b>B:</b> The prices, in rupees per unit, of the three commodities <math>X</math>, <math>Y</math> and <math>Z</math> are <math>x</math>, <math>y</math> and <math>z</math> respectively. <math>A</math> purchases 4 units of <math>Z</math> and sells 3 units of <math>X</math> and 5 units of <math>Y</math>. <math>B</math> purchases 3 units of <math>Y</math> and sells 2 units of <math>X</math> and 1 unit of <math>Z</math>. <math>C</math> purchases 1 unit of <math>X</math> and sells 4 units of <math>Y</math> and 6 units of <math>Z</math>.</p> <p>In the process <math>A</math>, <math>B</math> and <math>C</math> earn Rs.6000, Rs.5,000 and Rs.13,000 respectively. Using matrices, find the prices of the three commodities (note that selling the units is positive earning and buying the units is negative earning.)</p>	CO2

**(OR)**

**A:** Solve the following homogeneous system of equations for its non-trivial solution

$$x + 3y + 2z = 0, 2x - y + 3z = 0, 3x - 5y + 4z = 0, x + 17y + 4z = 0.$$

**B:** An amount of Rs. 4,000 is distributed into three investments at the rate of 7%, 8% and 9% per annum respectively. The total annual income is Rs. 317.50 and the annual income from the first investment is Rs. 5 more than the income from the second. Find the amount of each investment.