Name: **Enrolment No:**	UPES UNIVERSITY WITH A PURPOSE

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2020

Course: Group Theory — **Semester: V**
Program: B.Sc. (Hons.) Mathematics — **Time : 03 hrs.**
Course Code: MATH 3022 — **Max. Marks: 100**

Instructions: All questions are compulsory.

SECTION A

1. **Each question carries 5 marks.**
2. **Complete the statement / Select the correct answers(s).**

S. No.		
Q1	Which of the following numbers can be the possible order(s) of a permutation on 11 symbols which doesn't fix any symbol? a. 18 b. 30 c. 15 d. 28	**CO1**
Q2	Let $G = \mathbb{Z}_{10} \oplus \mathbb{Z}_{15}$. Then a. G contains exactly one element of order 2 b. G contains exactly 5 elements of order 3 c. G contains exactly 24 elements of order 5 d. G contains exactly 24 elements of order 10	**CO2**
Q3	Let G be a group of order 15. Then the number of Sylow subgroups of G of order 3 is ______	**CO5**
Q4	Which of the following can be a possible class equation of a group of order 10 ? a. $1+1+1+2+5=10$ b. $1+2+3+4=10$ c. $1+2+2+5=10$ d. $1+1+2+2+2+2=10$	**CO5**
Q5	The maximum order of an element in group $U(8)\oplus S_3\oplus\mathbb{Z}_{10}$ is ______	**CO3**
Q6	The set of all real 2×2 invertible matrices acts on $\mathbb{R}^2$ by matrix multiplication. The number of orbits for this group action is ____	**CO4**

SECTION B 1. **Each question carries 10 marks.** 2. **There is an internal choice in Q11.**		
Q7	Let $U(n)$ denotes the group of units in $\mathbb{Z}_n$. Show that $U(55)^3 = \{x^3 \vert x \in U(55)\}$ is $U(55)$.	**CO1**
Q8	Let $< \alpha >$ denotes the group generated by element α of group G. Find three cyclic subgroups of maximum possible order in $\mathbb{Z}_6 \oplus \mathbb{Z}_{10} \oplus \mathbb{Z}_{15}$ of the form $< a > \oplus < b > \oplus < c >$, where $a \in \mathbb{Z}_6, b \in \mathbb{Z}_{10}$ and $c \in \mathbb{Z}_{15}$.	**CO2**
Q9	Consider the symmetric group S_3 on symbols 1,2 and 3. If H and K are subgroups generated by the cycles (1 2 3) and (1 2) respectively. Prove or disapprove the following: $$S_3 = H \times K$$ where the symbol $' \times '$ represents the internal direct product.	**CO3**
Q10	Let G be a non-abelian group of order 343 such that the center $Z(G)$ is non-trivial. Prove that $\vert Z(G) \vert = 7$.	**CO3**
Q11	For an action of a group G on set A, state and prove Orbit-stabilizer theorem. **OR** For an action of a group G on set A, define the normalizer of set A as: $$N_G(A) = \{g \in G \vert gag^{-1} \in A \; \forall a \in A\}$$ Prove that $N_G(A_3) = A_3$, where A_3 is subgroup of all even permutations of symmetric group S_3.	**CO4**

SECTION-C 1. **Q12 carries (10+10) marks.** 2. **There is an internal choice in Q12.**		
Q 12	a. Let G be a finite group acting on itself by conjugation. If $g_i \in G$ and $Z(G)$ denotes the center of G then establish the result: $$\vert G \vert = \vert Z(G) \vert + \sum_{\{i=1\}}^{k} \vert G : C_G(g_i) \vert$$ where $C_G(x)$ is the centralizer of element x in G. b. If G be a non-abelian group of order 143. Determine its class equation. **OR** a. Consider the symmetric group S_4 on four symbols. Find all the conjugacy classes of S_4 by using the combinatorics on the cycle types in it. b. Use class equation to prove that the center of S_4 is trivial.	**CO4**