

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2020

Programme: B.Sc. (Hons.) Physics and Chemistry

Course Name: Differential Equations

Course Code: MATH 1034

No. of page/s: 02

Semester: III

Max. Marks: 100

Duration: 3 Hrs.

Section A

Attempt all the questions. This section contains 6 multiple-choice questions and one option is correct. Write the correct option. Each question carries 5 marks.

1.	<p>All real solutions of the differential equation $\frac{d^2y}{dx^2} + 2a\frac{dy}{dx} + by = \cos x$ (where a and b are real constants) are periodic if :</p> <p>A. $a = 1$ and $b = 0$ B. $a = 0$ and $b = 1$ C. $a = 1$ and $b \neq 0$ D. $a = 0$ and $b \neq 1$</p>	CO3
2.	<p>A particular solution of $4x^2\frac{d^2y}{dx^2} + 8x\frac{dy}{dx} + y = \frac{4}{\sqrt{x}}$ is:</p> <p>A. $\frac{1}{2\sqrt{x}}$ B. $\frac{\log x}{2\sqrt{x}}$ C. $\frac{(\log x)^2}{2\sqrt{x}}$ D. $\frac{\{(\log x)\sqrt{x}\}}{2}$</p>	CO3
3.	<p>Let the general integral of the partial differential equation $(2xy - 1)\frac{\partial z}{\partial x} + (z - 2x^2)\frac{\partial z}{\partial y} = 2(x - yz)$ be given by $F(u, v) = 0$, where $F: \mathbb{R}^2 \rightarrow \mathbb{R}$ is a continuously differentiable function. (\mathbb{R} is the set of all real numbers and $\mathbb{R}^2 = \{(x, y): x, y \in \mathbb{R}\}$). Which of the following is true?</p> <p>A. $u = x^2 + y^2 + z, v = xz + y$ B. $u = x^2 + y^2 - z, v = xz - y$ C. $u = x^2 - y^2 + z, v = yz + x$ D. $u = x^2 + y^2 - z, v = yz - x$</p>	CO5
4.	<p>The differential equation $(1 - x^2)\frac{\partial^2 z}{\partial x^2} - 2xy\frac{\partial^2 z}{\partial x \partial y} + (1 - y^2)\frac{\partial^2 z}{\partial y^2} + x\frac{\partial z}{\partial x} + 3x^2y\frac{\partial z}{\partial y} - 2z = 0$ is elliptic in the region:</p> <p>A. $x^2 + y^2 < 0$ B. $x^2 + y^2 < 1$ C. $x^2 + y^2 > 0$ D. $x^2 + y^2 > 1$</p>	CO1

5.	<p>The solution of $\frac{dy}{dt} - 3y = e^{2t}$, $y(0) = 1$ is:</p> <p>A. $2e^{2t} + e^{3t}$ B. $2e^{3t} + e^{2t}$ C. $2e^{3t} - e^{2t}$ D. $2e^{2t} - e^{3t}$</p>	CO2
6.	<p>The particular integral of the differential equation $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 5^x - \log_e 2$ is:</p> <p>A. $\frac{1}{(\log_e 5+3)^2} - \frac{1}{9}\log_e 2$ B. $\frac{1}{(\log_e 5+3)^2} 5^x - \frac{1}{9}\log_e 2$ C. $\frac{1}{(\log_e 5+3)^2} 5^x + \frac{1}{9}\log_e 2$ D. $\frac{1}{(\log_e 5+3)^2} 5^x$</p>	CO3
<p>SECTION B</p> <p>Attempt all the questions. This section contains descriptive type's questions. Each question carries 10 marks.</p>		
7.	<p>Form the partial differential equation by eliminating h and k from the equation $(x - h)^2 + (y - k)^2 = \lambda^2$.</p>	CO1
8.	<p>According to Newton's law of cooling, the rate at which a substance cools in moving air is proportional to the difference between the temperature of the substance and that of the air. If the temperature of the air is 290 K and the substance cools from 370 K to 330 K in 10 minutes, find time when the temperature will be 295 K.</p>	CO2
9.	<p>Solve $(D^2 + 3D + 2)y = e^{2x} \sin x$.</p>	CO3
10.	<p>Find $f(z)$ such that $\left[\frac{(y^2+z^2-x^2)}{2x}\right] dx - ydy + f(z)dz = 0$ is integrable. Hence solve it.</p>	CO4
11.	<p>Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$.</p>	CO3
<p>SECTION C</p> <p>This section contains descriptive type's question and it has internal choices. This question carries 20 marks.</p>		
12.	<p>Find the complete integral of $2(z + px + qy) = yp^2$ where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$.</p> <p style="text-align: center;">OR</p> <p>Solve $x(x^2 + 3y^2)\frac{\partial z}{\partial x} - y(3x^2 + y^2)\frac{\partial z}{\partial y} = 2z(y^2 - x^2)$.</p>	CO5