Name: Enrolment No:	UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, July 2020

Course: Mathematical Physics III **Semester: IV**
Course Code: PHYS 2004 **Time: 03 hrs.**
Programme: B.Sc. Physics (H) **Max. Marks: 100**

Instructions: Attempt all questions from **PART A** (60 Marks) and **PART B** (40 Marks). All questions are compulsory.

PART A

Instructions: PART A contains **25** questions for a total of 60 marks. It contains **22** multiple-choice questions and **3** multiple answer questions. Multiple answer questions may have more than one correct option. Select all the correct options. You need to answer **PART A** within the slot from **02:00** AM to **05:00** PM on **10th July 2020**. The due time for **PART A** is **05:00** PM on **10th July 2020**. After the due time, the **PART A** will not be available.

S. No.		Marks	CO
Q1 (i)	The modulus of the complex number $z=\frac{1+i}{1-i\sqrt{3}}$ is A. $\frac{1}{\sqrt{2}}$ B. $\sqrt{2}$ C. 0 D. 1	**2**	**CO1**
Q1 (ii)	The value of the integral $\int_C \frac{e^z}{z-2}dz$, where C is the circle $\lvert z\rvert=3$ is A. $\pi i e^2$ B. $2\pi i e^2$ C. $2ie^2$ D. $2\pi e^2$	**2**	**CO1**
Q1 (iii)	The nature of the singularity of the function $f(z)=\frac{z-\sin z}{z^3}$ at $z=0$ is A. *Removable Singularity* B. *Essential Singularity* C. *Pole* D. *None of these*	**2**	**CO1**

Q1 (iv)	The residue of the function $f(z) = \frac{z^2}{(z-1)(z-2)^2}$ at the pole $z = 1$ is given by A. 0 B. 1 C. 2 D. 3	**2**	**CO1**
Q1 (v)	Let $P(z) = a + bz$ and $\int_C \frac{P(z)}{z} dz = \int_C \frac{P(z)}{z^2} dz = 2\pi i$, where C is the circle $\|z\| = 1$. Then the value of the constants a and b is A. $a = 1, b = 1$ B. $a = 1, b = 2$ C. $a = 2, b = 1$ D. $a = 2, b = 2$	**2**	**CO1**
Q1 (vi)	Laplace transform of $t^3 e^{-3t}$ is A. $\frac{7}{(s+4)^3}$ B. $\frac{s}{(s+3)^3}$ C. $\frac{6}{(s+3)^4}$ D. $\frac{2}{(s+6)^3}$	**2**	**CO2**
Q1 (vii)	$L[2 \sin 2t \cos 4t] =$…………… A. $\frac{3}{(s^2+36)} - \frac{2}{(s^2+4)}$ B. $\frac{6}{(s^2+36)} + \frac{2}{(s^2+4)}$ C. $\frac{6}{(s^2+36)} - \frac{1}{(s^2+4)}$ D. $\frac{6}{(s^2+36)} - \frac{2}{(s^2+4)}$	**2**	**CO2**
Q1 (viii)	If $F(s)$ is the Fourier transform of $f(x)$, then the Fourier transform of $f(x-a)$ is A. $-e^{ias} F(s)$. B. $e^{-ias} F(s)$. C. $e^{ias} F(s)$. D. $None\ of\ these$.	**2**	**CO2**

Q1 (ix)	Let $f(x)$ be a function defined on $(-\infty,\infty)$ and is piecewise continuous, differentiable in each finite interval and is absolutely integrable on $(-\infty,\infty)$. Then Fourier transform of $f(x)$ is given by, A. $F[f(x)] = F(s) = \frac{1}{\sqrt{2}}\int_{-\infty}^{\infty} f(x)e^{-isx}dx.$ B. $F[f(x)] = F(s) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{isx}dx.$ C. $F[f(x)] = F(s) = \frac{1}{\sqrt{2}}\int_{-\infty}^{\infty} f(x)e^{isx}dx.$ D. $F[f(x)] = F(s) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty} f(x)e^{isx}dx$	**2**	**CO2**
Q1 (x)	The Laplace transform of the piecewise continuous function $f(t) = \begin{cases} 0, & 0 \le t < 2 \\ k, & t \ge 2 \end{cases}$; ($k$ is a constant) is A. $\frac{ke^{-2s}}{s}$ $(s > 0)$ B. $\frac{e^{-2s}}{s}$ $(s > 0)$ C. $\frac{ke^{2s}}{s}$ $(s > 0)$ D. $\frac{ke^{-s}}{s}$ $(s > 0)$	**2**	**CO3**
Q1 (xi)	The Fourier sine transform of e^{-x} is A. $\sqrt{\frac{2}{\pi}}\left(\frac{s}{1+s^2}\right).$ B. $\sqrt{\frac{2}{\pi}}\left(\frac{1}{1-s^2}\right).$ C. $\sqrt{\frac{2}{\pi}}\left(\frac{1}{1+s^2}\right).$ D. $\sqrt{\frac{2}{\pi}}\left(\frac{s}{1-s^2}\right).$	**2**	**CO3**
Q1 (xii)	The Laplace transform of a unit step function with step at point 'a' is given by A. e^{-as} B. $\frac{1}{s}e^{-as}$ C. $-\frac{1}{s}e^{-as}$ D. $-\frac{1}{s}e^{as}$	**2**	**CO3**

Q1 (xiii)	If $F(s)$ is the Fourier transform of $f(x)$, then the Fourier transform of $f(x)\cos ax$ is A. $\frac{1}{2}[F(s-a)-F(s+a)]$. B. $\frac{1}{4}[F(s-a)-F(s+a)]$. C. $\frac{1}{2}[F(s-a)+F(s+a)]$. D. $\frac{1}{4}[F(s-a)+F(s+a)]$.	**2**	**CO4**
Q1 (xiv)	The value of the integral $\int_0^\infty e^{-3t}\,\delta(t-4)dt$ is A. e^{-12s} B. e^{12s} C. e^{-12} D. e^{12}	**2**	**CO4**
Q1 (xv)	If $F(s)$ and $G(s)$ are the Fourier transform of $f(x)$ and $g(x)$ respectively and c_1 and c_2 are constants, then A. $F^{-1}[c_1F(s)+c_2G(s)] = c_1\,f(x).c_2\,g(x)$ B. $F^{-1}[c_1F(s)+c_2G(s)] = c_1\,f^{-1}(x).c_2\,g^{-1}(x)$ C. $F^{-1}[c_1F(s)+c_2G(s)] = c_1\,f^{-1}(x)+c_2\,g^{-1}(x)$ D. $F^{-1}[c_1F(s)+c_2G(s)] = c_1\,f(x)+c_2\,g(x)$	**2**	**CO4**
Q1 (xvi)	The residue of the function $f(z)=\frac{z^2-2z}{(z^2+4)(z+1)^2}$ at the pole $z=-1$ is given by A. $-11/25$ B. $-12/25$ C. $-13/25$ D. $-14/25$	**3**	**CO1**
Q1 (xvii)	The Laurent's series expansion of the function $f(z)=\frac{1}{(z+2)(1+z^2)}$ valid in the region $1<\|z\|<2$ is A. $f(z)=\frac{1}{10}\sum_{n=0}^{\infty}(-1)^n\left(\frac{z}{2}\right)^n+\frac{2-z}{5z^2}\sum_{n=0}^{\infty}(-1)^n\left(\frac{1}{2z^2}\right)^n$ B. $f(z)=\frac{1}{10}\sum_{n=0}^{\infty}(-1)^n\left(\frac{z}{2}\right)^n+\frac{2-z}{5z^2}\sum_{n=0}^{\infty}(-1)^n\left(\frac{1}{z^2}\right)^n$ C. $f(z)=\frac{1}{10}\sum_{n=0}^{\infty}(-1)^n\left(\frac{z}{2z}\right)^n+\frac{2-z}{5z^2}\sum_{n=0}^{\infty}(-1)^n\left(\frac{1}{z^2}\right)^n$ D. $f(z)=\frac{1}{10}\sum_{n=0}^{\infty}(-1)^n\left(\frac{z}{3}\right)^n+\frac{2-z}{5z^2}\sum_{n=0}^{\infty}(-1)^n\left(\frac{1}{z^2}\right)^n$	**3**	**CO1**

Q1 (xviii)	In the Laurent's series expansion of $f(z) = \frac{1}{e^z - 1}$ about the point $z = 0$ valid in the region $0 < \lvert z \rvert < 2\pi$, which are the following are true? (More than one answer can be correct) A. $coefficient\ of\ 1/z\ is\ 1$ B. $coefficient\ of\ 1/z\ is\ 0$ C. $coefficient\ of\ z\ is\ 1/12$ D. $coefficient\ of\ z\ is\ 1/2$	**3**	**CO1**
Q1 (xix)	The function whose Laplace transform is $\tan^{-1}(1+s)$; given as A. $-\frac{1}{t}e^{-t}\sin t$ B. $\frac{1}{t}e^{-t}\sin t$ C. $-\frac{1}{t}e^{t}\sin t$ D. $\frac{1}{t}e^{t}\sin t$	**3**	**CO2**
Q1 (xx)	If $L[f(t)] = F(s)$, then which of the following statements are correct. (More than one answer can be correct) A. $L\left[\int_0^t f(t)dt\right] = \frac{1}{s}F(s)$ B. $L[f'(t)] = sL[f(t)] - f(0)$ C. $L[t^n f(t)] = \frac{d^n}{ds^n}F(s)$ D. All of the above	**3**	**CO2**
Q1 (xxi)	The value of the integral $\int_0^\infty te^{-3t}\sin t\, dt$ is A. $\frac{1}{50}$ B. 0 C. -1 D. $None\ of\ these$	**3**	**CO3**
Q1 (xxii)	The Fourier Transform of the function $$f(x) = e^{-a\lvert x \rvert};\ -\infty < x < \infty$$ is given by A. $\sqrt{\frac{2}{\pi}}\left(\frac{a}{a^2+s^2}\right).$ B. $\sqrt{\frac{2}{\pi}}\left(\frac{a}{a^2-s^2}\right).$	**3**	**CO3**

	C. $\sqrt{\frac{1}{2\pi}}\left(\frac{a}{a^2+s^2}\right)$. D. $\sqrt{\frac{1}{2\pi}}\left(\frac{a}{a^2-s^2}\right)$.		
Q1 (xxiii)	The Fourier Transform of the function $$f(x)=\begin{cases}1\ , & \lvert x\rvert\le a\\ 0\ , & \lvert x\rvert > a\end{cases}$$ is given by (More than one answer can be correct) A. $-\frac{i}{s\sqrt{2\pi}}\left(e^{ias}-e^{-ias}\right)$. B. $\frac{i}{s\sqrt{2\pi}}\left(e^{ias}-e^{-ias}\right)$. C. $\sqrt{\frac{2}{\pi}}\left(\frac{\sin sa}{s}\right)$ D. $\sqrt{\frac{2}{\pi}}\left(\frac{\cos sa}{s}\right)$	**3**	**CO3**
Q1 (xxiv)	If $F(s)=\frac{1}{(2s+3)^{1/2}}$ then $L^{-1}[F(s)]=$………. A. $\frac{1}{\sqrt{2\pi}}t^{-1/2}e^{3t/2}$ B. $\frac{1}{\sqrt{2\pi}}t^{-1/2}e^{-3t/2}$ C. $\frac{1}{\sqrt{2\pi}}t^{1/2}e^{-3t/2}$ D. $\frac{1}{\sqrt{2\pi}}t^{1/2}e^{3t/2}$	**3**	**CO4**
Q1 (xxv)	The Fourier cosine transform of $$f(x)=\begin{cases}x, & 0<x<1\\ 2-x, & 1<x<2\\ 0, & x>2\end{cases}$$ is A. $\sqrt{\frac{8}{\pi}}\left\{\frac{\cos s\,(1+\cos s)}{s^2}\right\}$. B. $\sqrt{\frac{8}{\pi}}\left\{\frac{\cos s\,(1-\cos s)}{s^2}\right\}$. C. $\sqrt{\frac{4}{\pi}}\left\{\frac{\cos s\,(1-\cos s)}{s^2}\right\}$. D. $\sqrt{\frac{4}{\pi}}\left\{\frac{\cos s\,(1+\cos s)}{s^2}\right\}$.	**3**	**CO4**

PART B **Instructions:** The link for PART B will be available from 10:00 AM on 10th July 2020 to 10:00 AM on 11th July 2020. Solve the problems in PART B on a plain A4 sheets and write your name, roll number and SAP ID on each page and then scan them into a single PDF file. Name the file as SAP ID _BRANCH NAME_ROLL NUMBER (for example: 500077624_CCVT_ R103219023.pdf) and upload that PDF file through the link provided over there. PART B solutions sent through WhatsApp or email will not be entertained.			
Q2	Evaluate $\int_0^\infty \frac{1}{(x^4+1)} dx$ using complex integration.	**8**	**CO1**
Q3	Find the Laplace transform of $f(t) = \frac{\cos\sqrt{t}}{\sqrt{t}}$.	**8**	**CO2**
Q4	Find the Fourier Transform of the function $$f(x) = \begin{cases} 1\ , & \lvert x\rvert \le a \\ 0\ , & \lvert x\rvert > a \end{cases}$$ and hence evaluate $$\int_{-\infty}^{\infty} \frac{\sin sa \cos sx}{s} ds \ \ and \int_0^{\infty} \frac{\sin sa}{s} ds$$	**8**	**CO3**
Q5	Solve the heat conduction problem described by $$PDE: k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t},\ \ 0 < x < \infty, t > 0,$$ $$BC: \quad u(0,t) = u_{0,} \quad\quad t \ge 0,$$ $$IC: \quad u(x,0) = 0, \quad 0 < x < \infty,$$ u and $\partial u/\partial x$ both tend to zero as $x \to \infty$.	**8**	**CO4**
Q6	Solve the following differential equation using Laplace transform $$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 8y = \cos 2t; y(0) = 2, y'(0) = 1.$$	**8**	**CO4**