Name: **Enrolment No:**	UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, May 2020

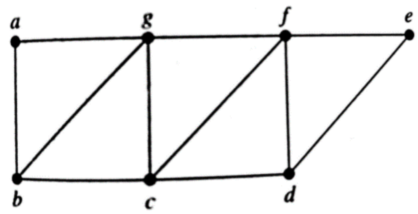
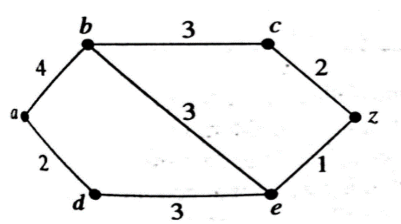
Course: Graph Theory | **Semester:** IV
Course Code: MATH 2025 | **Time: 03 hrs.**
Programme: B.Sc.(Hons.) Mathematics | **Max. Marks: 100**

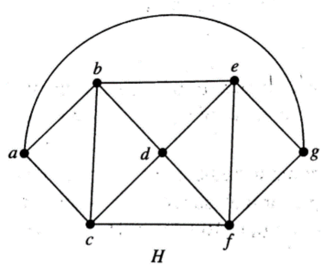
Instructions: Attempt all questions from **PART A** (60 Marks) and **PART B** (40 Marks). All questions are compulsory.

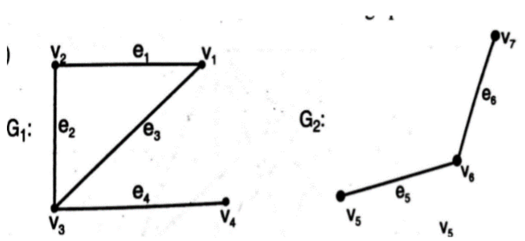
PART A

Instructions: PART A contains 25 questions for a total of 60 marks. It contains 20 multiple choice questions and 5 multiple answer questions. Multiple answer questions may have more than one correct option. Select all the correct options. You need to answer PART A within the slot from 10:00 AM to 1:00 PM on 6th July 2020. The due time for PART A is 1:00 PM on 6th July 2020. After the due time, the PART A will not be available.

S. No.		Marks	CO
Q1	Maximum number of edges in a simple graph with n vertices is: A. n B. $\frac{n(n+1)}{2}$ C. $\frac{n(n-1)}{2}$ D. $2n$	2	CO1
Q2	Number of edges in 4-regular graph with six vertices are: A. 4 B. 6 C. 12 D. 10	2	CO1
Q3	Number of edges in a bipartite graph with n vertices is at max: A. $\frac{n}{2}$ B. $\frac{n^2}{2}$ C. $\frac{n}{4}$ D. $\frac{n^2}{4}$	2	CO2
Q4	How many edges are there in a graph with 10 vertices each of degree 6? A. 30	2	CO2

	B. 16 C. 40 D. 10		
Q5	Euler Path in the following graph is given by:  A. $b, a, g, f, d, e, f, c, g, b$ B. $b, a, g, f, e, d, c, g, b, c, f, d$ C. $b, a, g, c, f, d, e, f, c, b$ D. $b, g, c, f, d, e, f, c, g, a$	**2**	**CO2**
Q6	K_n has a Hamiltonian circuit when: A. $n \geq 3$ B. $n \leq 3$ C. It doesn't depends on n D. $n \geq 2$	**2**	**CO2**
Q7	The shortest path between a and z in the graph:  A. a, b, c, z B. a, b, e, z C. a, d, e, b, c, z D. a, d, e, z	**3**	**CO3**
Q8	In a connected planar simple graph with 20 vertices each of degree 3. In how many regions does a representation of this planar graph split the plane? A. 10 B. 12 C. 20 D. 30	**3**	**CO2**

<table>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>Q9</td><td>Chromatic number of the graph H is given by:


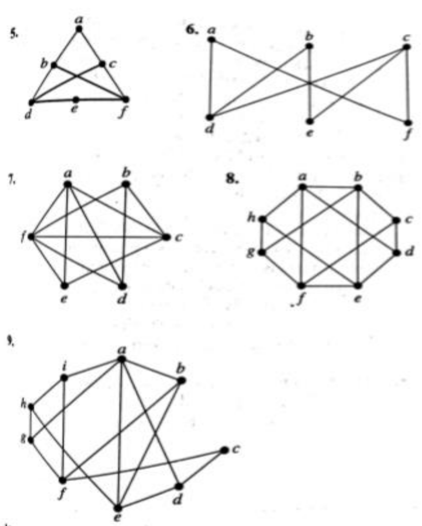
H
A. 5
B. 4
C. 3
D. 6</td><td>3</td><td>CO4</td></tr>
<tr><td>Q10</td><td>The incidence matrix of the following disconnected graphs G_1 and G_2 is given by:


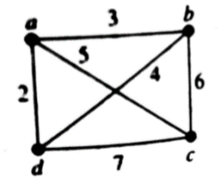
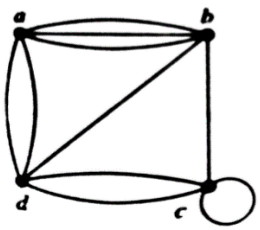
A. $\begin{matrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{matrix}$

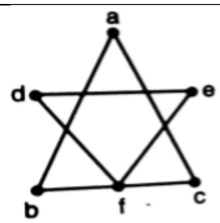
B. $\begin{matrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{matrix}$

C. $\begin{matrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{matrix}$</td><td>3</td><td>CO3</td></tr>
</table>

	$\begin{matrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{matrix}$ D. $\begin{matrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{matrix}$		
Q11	How many vertices and how many edges do the $K_{m,n}$ have? A. m, n B. $m-n, m+n$ C. $m+1, mn$ D. $\frac{m(n+1)}{2}$	**2**	**CO2**
Q12	If the simple graph G has v vertices and e edges, how many edges does G' (Complement of G) Have? A. $\frac{v(v-1)}{2} - e$ B. $\frac{v(v-1)}{2} + e$ C. $\frac{v(v-1)}{2}$ D. $\frac{v(v+1)}{2} - e$	**2**	**CO2**
Q13	The Chromatic number of Peterson graph is: A. 2 B. 3 C. 4 D. 5	**2**	**CO4**
Q14	Let G be a single connected graph with $n \geq 3$ vertices. If $\deg(v) + \deg(w) \geq n$ for each vertices v and w, not connected by an edge, then which statement is correct: A. G is Eulerian B. G is Hamiltonian C. G is Planar D. G is dual	**2**	**CO3**

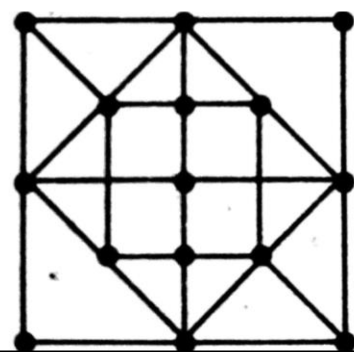
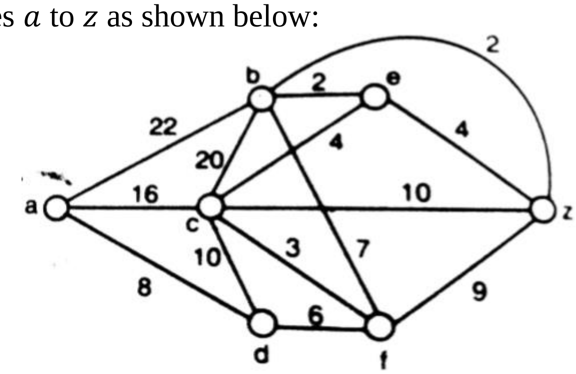
Q15	If G is a connected graph other than, complete graph with ΔG (maximum degree of a vertex) then the relation between ΔG and $\chi(G)$ is: A. $\chi(G) \geq \Delta(G)$ B. $\Delta(G) \leq \chi(G)$ C. $\chi(G) \leq \Delta(G)$ D. $\Delta(G) \geq \chi(G)$	**2**	**CO5**
Q16	The graph G is self-complementary if it has: A. $4n$ vertices B. $4n-1$ vertices C. $4n+1$ vertices D. None of these	**3**	**CO2**
Q17	If a connected planner simple graph has e edges and vertices with $v \geq 3$ and no circuits of length three, then A. $e \geq 2v-4$ B. $e \leq 2v-4$ C. $e \leq v-2$ D. $e \leq v+4$	**3**	**CO3**
Q18	In figures, 5-9 determine which of the following graphs are planar:  A. 8 B. 7, 6, 9 C. 7 D. 5, 6, 7	**3**	**CO2**

<table>
<tr><td>Q19</td><td>Solution of the traveling salesman problem for the following graph is:

A. a, d, c, b, a
B. a, c, b, d, a
C. b, c, d, a, b
D. c, d, a, b, c</td><td>2</td><td>CO3</td></tr>
<tr><td>Q20</td><td>The adjacency matrix to represent the following Pseudo graph is:

A. $\begin{matrix} 0 & 2 & 0 & 2 \\ 3 & 0 & 1 & 1 \\ 1 & 1 & 0 & 2 \\ 2 & 1 & 1 & 0 \end{matrix}$
B. $\begin{matrix} 0 & 2 & 0 & 2 \\ 2 & 0 & 1 & 1 \\ 1 & 1 & 0 & 2 \\ 2 & 1 & 1 & 2 \end{matrix}$
C. $\begin{matrix} 0 & 3 & 0 & 2 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 2 & 0 \end{matrix}$
D. $\begin{matrix} 0 & 2 & 0 & 2 \\ 3 & 0 & 3 & 1 \\ 1 & 1 & 0 & 2 \\ 2 & 1 & 1 & 0 \end{matrix}$</td><td>3</td><td>CO3</td></tr>
<tr><td>Q21</td><td>The number of connected components in the following graphs are:</td><td>2</td><td>CO2</td></tr>
</table>

	 A. 1 B. 2 C. 3 D. 0		
Q22	19. A planar graph G is said to be self-dual if: A. G is complement to its dual G B. G is homomorphic to its dual G C. G is isomorphic to its dual G D. None of these	**2**	**CO3**
Q23	The minimum number of colors required in an edge coloring of G is known as: A. Chromatic number B. Chromatic polynomial C. Chromatic index D. Edge coloring	**2**	**CO5**
Q24	20. Two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are said to be isomorphic if : A. If there exists a function $f: V_1 \to V_2$ such that f is one-to-one into B. If there exists a function $f: V_1 \to V_2$ such that f is one-to-one onto C. $\{a, b\}$ is an edge in E_1, if and only if $\{f(a), f(b)\}$ is an edge in E_2 for any two elements $a, b \in V_1$ D. The function f doesn't preserve adjacency	**3**	**CO2**
Q25	21. Identify the correct statements associated with a graph: A. A walk is a trail if all its edges are distinct. B. A walk is called a path if all its vertices and edges are same. C. A walk is called a path if all its vertices and edges are distinct. D. A path in which two-repeated vertices are allowed is known as cycle.	**3**	**CO3**

PART B

The link for PART B will be available from 2:00 PM on 12th July 2020 to 2:00 PM on 13th July 2020. Solve the problems in PART B on a plain A4 sheets and write your name, roll number and SAP ID on each page and then scan them into a single PDF file. Name the file as SAP ID _BRANCH NAME_ROLL NUMBER (for example: 500077624_CCVT_ R103219023.pdf) and upload that PDF file through the link provided over there. PART B solutions sent through WhatsApp or email will not be entertained.

Q 1	If a connected planar graph G has n vertices, e edges and r region, then by induction show that $n - e + r = 2$	CO6	8
Q2.	Determine the number of vertices, the number of edges, and the number of region in the graph shown below. Then show that your answer satisfy the Euler's theorem for connected planar graphs. 	CO3	6
Q3.	Apply Dijksta's algorithm to the graph given below to determine the shortest path between the vertices a to z as shown below:  	CO3	8
Q 4	Construct an influence graph for the board members of a company if the President can influence the Director of Research and Development, The Director of Marketing, and the Director of Operations; the Director of Research and Development can influence the Director of Operations; the Director of Marketing can influence the Director of Operations; and no one can influence, or be influenced by, the Chief Financial officer.	CO1	6
Q5(a)	Q5 (a). Seven courses $C_1, C_2 \dots \dots, C_7$ are to be scheduled at a university examinations, where the following pairs of courses have common student $(C_1, C_2);\ (C_1, C_3), (C_1, C_4), (C_1, C_7), (C_2, C_3), (C_2, C_4), (C_2, C_5), (C_2, C_7), (C_3, C_4), (C_3, C_6$ $(C_3, C_7), (C_4, C_5), (C_4, C_6), (C_5, C_6), (C_5, C_7)\ and\ \ (C_6, C_7)$ How can the examination be scheduled so that no students has two examination at the same time?	CO5	6

(b)	Explain the regular graph with the help of an example also find the size of an r $-$regular (p, q) graph and hence find whether there exists a 4 $-$regular graph on 6 vertices. If so construct a graph.	CO2	6