Name: **Enrolment No:**	UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, May 2020

Course: Differential Equations **Semester: II**
Course Code: MATH 1031 **Time: 03 hrs.**
Programme: B.Sc. (H) Mathematics **Max. Marks: 100**
Instructions: Attempt all questions from **PART A** (60 Marks) and **PART B** (40 Marks). All questions are compulsory.

PART A

Instructions: PART A contains 25 questions for a total of 60 marks. It contains 22 multiple choice questions and 3 multiple answer questions. Multiple answer questions may have more than one correct option. Select all the correct options. You need to answer PART A within the slot from 10:00 AM to 1:00 PM on 8th July 2020. The due time for PART A is 1:00 PM on 8th July 2020. After the due time, the PART A will not be available.

S. No.		**Marks**	**CO**
Q1 (i)	The solution of differential equation $(x+1)\frac{dy}{dx} = x(y^2+1)$ is given by A. $\tan^{-1} x = y - \log(1+y) + c$ B. $\tan^{-1} y = x - \log(1+x) + c$ C. $\tan^{-1} x = y + \log(1+y) + c$ D. $\tan^{-1} y = x - \log(1+y) + c$	**2**	**CO1**
Q1 (ii)	Find the value of λ for which the differential equation $(xy^2 + \lambda x^3y^2)dx + (x^3y + yx)x\,dy = 0$ is exact. A. $\lambda = 2$ B. $\lambda = -2$ C. $\lambda = 1$ D. $\lambda = 0$	**2**	**CO2**
Q1 (iii)	The solution of $(x+1)\frac{dy}{dx} - y = e^x(x+1)^2$ is given by A. $\frac{y}{x+1} = e^x + c$ B. $y = (x+1)e^{xc}$ C. $y = (x-1)e^{-x} + c$ D. None of these	**3**	**CO2**
Q1 (iv)	The solution of $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$ is given by	**2**	**CO3**

	A. $y = e^{-2x}(A\cos x + B\sin x)$ B. $y = e^{2x}(A\cos x + B\sin x)$ C. $y = e^{-x}(A\cos 2x + B\sin 2x)$ D. $y = e^{x}(A\cos 2x + B\sin 2x)$		
Q1 (v)	The particular integral of $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 6e^{3x}$ is given by A. $\frac{1}{2}x^2e^{3x}$ B. $3x^2e^{3x}$ C. $\frac{1}{2}xe^{3x}$ D. Not defined	**3**	**CO3**
Q1 (vi)	The solution of $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = 0$ is given by A. $y = C_1e^{-x} + C_2e^{4x}$ B. $y = C_1e^{-z} + C_2e^{4z}$ C. $y = \frac{C_1}{x} + C_2x^4$ D. $y = C_1x + \frac{C_2}{x^4}$	**3**	**CO3**
Q1 (vii)	$y = e^{-x}$ is a part of complementary function of differential equation $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$. If A. $1 + P + Q = 0$ B. $1 - P - Q = 0$ C. $1 - P + Q = 0$ D. $1 + P - Q = 0$	**2**	**CO3**
Q1 (viii)	Normal form of the second order differential equation $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$ is given by $\frac{d^2v}{dx^2} + Av = B$ where A and B are given by A. $A = Q + \frac{1}{2}\frac{dP}{dx} + \frac{P^2}{4}, B = \frac{R}{U}$ B. $A = Q + \frac{1}{2}\frac{dP}{dx} - \frac{P^2}{4}, B = \frac{R}{U}$ C. $A = Q - \frac{1}{2}\frac{dP}{dx} + \frac{P^2}{4}, B = \frac{R}{U}$ D. $A = Q - \frac{1}{2}\frac{dP}{dx} - \frac{P^2}{4}, B = \frac{R}{U}$	**2**	**CO3**
Q1 (ix)	Which of the following equation represents an exponential decay? A. $N = n_0e^{-kt}$ B. $N = n_0e^{kt}$	**2**	**CO4**

	C. $N = -n_0 e^{-kt}$ D. $N = -n_0 e^{kt}$		
Q1 (x)	Maximum number of individuals, an environment can support is known as A. Density growth model B. Harvesting C. Carrying capacity D. None of these	**2**	**CO4**
Q1 (xi)	If x is population at any time t and k is carrying capacity of population, then logistic equation is given by A. $\frac{dx}{dt} = rx\left(1 - \frac{x}{k}\right)$ B. $\frac{dx}{dt} = r\left(1 - \frac{x}{k}\right)$ C. $\frac{dx}{dt} = rx\left(1 + \frac{x}{k}\right)$ D. $\frac{dx}{dt} = r\left(1 + \frac{x}{k}\right)$	**2**	**CO4**
Q1 (xii)	In lake pollution model, we take the following assumptions **(check all correct answers)** A. Pollution are well mixed in the lake. B. Lake has constant volume. C. The flow of mixture into lake is equal to flow of mixture out of the lake. D. Pollution may collect in any part of the lake.	**3**	**CO4**
Q1 (xiii)	The number of bacteria in a culture is growing at a rate of $3000e^{2t/5}$ per unit of time t. At $t = 0$, the number of bacteria present was 7,500. Find the number present at $t = 5$. A. $\cong 55418$ B. $\cong 60418$ C. $\cong 65418$ D. $\cong 70418$	**3**	**CO4**

Q1 (xiv)	A slow economy caused a company's annual revenues to drop from \$ 530,000 in 2008 to \$ 386,000 in 2010. If the revenue is following an exponential pattern of decline, what is the expected revenue in 2012? A. \$ 81,124 B. \$ 181,124 C. \$ 381,124 D. \$ 281,124	**3**	**CO4**
Q1 (xv)	If C_{in} is concentration of pollution of incoming water, C_0 is concentration of pollution in the lake at time $t = 0$, F is volume of water flowing in and out of the lake and V is the volume of lake then Concentration of pollutant in lake at any time 't' is given by A. $C(t) = C_{in} + e^{-\frac{F}{V}t}(C_0 - C_{in})$ B. $C(t) = C_{in} - e^{-\frac{F}{V}t}(C_{in} - C_0)$ C. $C(t) = C_{in} - C_{in}e^{-\frac{F}{V}t} + C_0 e^{-\frac{F}{V}t}$ All are correct	**2**	**CO4**
Q1 (xvi)	The time required for a quantity to reduce to half its initial value is known as A. Double life B. Decay C. Life D. Half life	**2**	**CO4**
Q1 (xvii)	In Drug Assimilation Model, we take following compartment A. Liver and blood B. GI-Tract and Blood C. Kidney and blood D. GI-Tract and Kidney	**2**	**CO4**
Q1 (xviii)	Given that, the initial population is 100. Suppose the population can be modelled using the differential equation $\frac{dx}{dt} = 0.2x - 0.001\, x^2$ with a time step of one month. Find the value of r, k and x_0 in logistic equation A. $r = 0.2, k = -200, x_0 = 100$ B. $r = -0.2, k = -0.001, x_0 = 100$ C. $r = 0.2, k = 0.001, x_0 = 100$	**3**	**CO4**

	D. $r = 0.2, k = 200, x_0 = 100$		
Q1 (xix)	The equilibrium points of the system $\frac{dx}{dt} = 3x - 2xy, \frac{dy}{dt} = xy - y$ are A. $(1,0)\ and\ \left(0, \frac{3}{2}\right)$ B. $(0,1)\ and\ \left(1, \frac{3}{2}\right)$ C. $(0,0)\ and\ \left(1, \frac{3}{2}\right)$ $(0,1)\ and\ \left(0, \frac{3}{2}\right)$	**3**	**CO5**
Q1 (xx)	Time between infection and the ability to infect someone else with the disease is known as A. Incubation Period B. Latent Period C. A & B are correct D. A & B are wrong	**2**	**CO5**
Q1 (xxi)	The system of equations of Predator-Prey Model is also known as A. Lotka-Volterra Model B. SIR Model C. A & B are correct D. A & B are wrong	**2**	**CO5**
Q1 (xxii)	The lines which divides phase plane in different regions where the directions of trajectories are different are known as A. Nullcurves B. Tangent line C. Asymptote D. Nullclines	**2**	**CO5**
Q1 (xxiii)	Those people who are infected from disease and are capable of spreading it to others are known as A. Contagious infectives B. Recovered C. Susceptibles	**2**	**CO5**

	None of these		
Q1 (xxiv)	In Epidemic Model of Influenza, the word equation of infectives compartment is given by A. $\begin{Bmatrix} Rate\ of\ change \\ of\ infectives \end{Bmatrix} = \begin{Bmatrix} rate\ of \\ infectives \\ recovered \end{Bmatrix} - \begin{Bmatrix} Rate\ of \\ susceptibles \\ infected \end{Bmatrix}$ B. $\begin{Bmatrix} Rate\ of\ change \\ of\ infectives \end{Bmatrix} = \begin{Bmatrix} rate\ of \\ infectives \\ recovered \end{Bmatrix}$ C. $\begin{Bmatrix} Rate\ of\ change \\ of\ infectives \end{Bmatrix} = -\begin{Bmatrix} Rate\ of \\ susceptibles \\ infected \end{Bmatrix}$ D. $\begin{Bmatrix} Rate\ of\ change \\ of\ infectives \end{Bmatrix} = \begin{Bmatrix} Rate\ of \\ susceptibles \\ infected \end{Bmatrix} - \begin{Bmatrix} rate\ of \\ infectives \\ recovered \end{Bmatrix}$	**3**	**CO5**
Q1 (xxv)	In Predator-Prey Model, If x is number of prey per unit area, y is number of predator per unit area, a_1 is per capita natural death rate of prey, b_1 is per capita birth rate of prey and others are constant then the differential equation of **Prey compartment** is given by **(Check all correct answers)** A. $\frac{dx}{dt} = (b_1 - a_1)x - c_1xy$ B. $\frac{dx}{dt} = \lambda_1 x - c_1xy$ C. $\frac{dx}{dt} = b_2y + kc_1xy - a_2y$ D. $\frac{dx}{dt} = c_2xy - \lambda_2 y$	**3**	**CO5**

PART B

The link for PART B will be available from 10:00 AM on 8th July 2020 to 10:00 AM on 9th July 2020. Solve the problems in PART B on a plain A4 sheets and write your name, roll number and SAP ID on each page and then scan them into a single PDF file. Name the file as SAP ID _BSC_MATH_ROLL NUMBER (for example: 500077624_BSC_MATH_ R103219023.pdf) and upload that PDF file through the link provided over there. PART B solutions sent through WhatsApp or email will not be entertained.

Q2 (A)	An integrating factor of the following equation is of the form y^n. Find n and hence solve the equation $y\sec^2 x\, dx + \left[3\tan x - \left(\frac{\sec y}{y}\right)^2\right] dy = 0.$	**4**	**CO2**

Q2 (B)	Find the nature of solution of the differential equation $\frac{dy}{dx} = \frac{x^2}{1+y^2}$.	**4**	**CO1**
Q3	Prove that the complete solution of differential equation $(D^2 - 1)y = \cosh x \cos x$ is $y = C_1 e^x + C_2 e^{-x} + \frac{2}{5}\sin x \sinh x - \frac{1}{5}\cos x \cosh x$.	**8**	**CO3**
Q4	Given that the developed model to describe the levels of antihistamine and decongestant in a patient taking a course of cold pills is $$\frac{dx}{dt} = I - k_1 x, \qquad x(0) = 0,$$ $$\frac{dy}{dt} = k_1 x - k_2 y, \qquad y(0) = 0.$$ Here k_1 and k_2 describe rates at which the drugs move between the two compartments (the GI-tract and the bloodstream) and I denotes the amount of drug released into the GI-tract in each time step. The levels of the drug in the GI-tract and the bloodstream are x and y respectively. By solving the equations sequentially show that the solution is $x(t) = \frac{I}{k_1}(1 - e^{-k_1 t}), y(t) = \frac{I}{k_2}\left(1 - \frac{1}{k_2 - k_1}(k_2 e^{-k_1 t} - k_1 e^{-k_2 t})\right)$	**8**	**CO4**
Q5	Let in a lake, the pollution level is 7%. If the concentration of incoming water is 2% and 10000 litres per day water is allowed to enter the lake, find time when pollution level is 5%. Volume of the lake is 200000 litres. Also, find the pollution after 32 days.	**8**	**CO4**
Q6	Consider the aimed fire battle model $$\frac{dR}{dt} = -a_1 B, \qquad \frac{dB}{dt} = -a_2 R.$$ Find the exact solution using theoretical techniques as follows: **(a)** Take the derivative of the first equation to get second-order differential equation and then eliminate $\frac{dB}{dt}$ from this equation by substituting the second equation into this second-order differential equation.	**8**	**CO5**

	(b) Now assume the solution to be an exponential of the form $e^{\lambda t}$. Substitute it into the second-order equation and solve for the two possible values of λ. The general solution for R will be of the form $R(t) = c_1 e^{\lambda_1} + c_2 e^{\lambda_2}$, where c_1 and c_2 are the arbitrary constants of integration. The solution for B is then found using the equation $\frac{dR}{dt} = -a_1 B$. **(c)** Now find the arbitrary constants by applying the initial conditions $R(0) = r_0$ and $B(0) = b_0$, when $t = 0$.		