Name: Enrolment No:	UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, July 2020

Course: Statistics, Numerical Methods & Algorithms **Semester: II**
Course Code: MATH 1025 **Time: 03 hrs.**
Programme: BCA. (IOT+BFSI) **Max. Marks: 100**
Instructions: Attempt all questions from **PART A** (60 Marks) and **PART B** (40 Marks). All questions are compulsory.

PART A

Instructions: PART A contains 25 questions for a total of 60 marks. It contains 25 multiple-choice questions. You need to answer PART A within the slot from 10:00 AM to 1:00 PM on 6th July 2020. The due time for PART A is 1:00 PM on 6th July 2020. After the due time, the PART A will not be available.

S. No.		Marks	CO
Q1 (i)	If 625.483 is approximated to three significant digit, then the percentage error will be: A. 0.483% B. 0.000772% C. 0.00772% D. 0.0772%	**2**	**CO1**
Q1 (ii)	If $\sqrt{29} = 5.385$ is correct to four significant digit, then the maximum error in this case will be: A. $\frac{1}{2} \times 10^{-3}$ B. $\frac{1}{2} \times 10^{3}$ C. $\frac{1}{2} \times 10^{-4}$ D. $\frac{1}{2} \times 10^{4}$	**2**	**CO1**
Q1 (iii)	The interval of the root of the algebraic equation $x^3 - 4x - 9 = 0$ is: A. (0,1) B. (1,2) C. (2,3) D. (3,4)	**2**	**CO1**

Q1 (iv)	In fixed point iteration method, which of the following condition is true: A. $\|\phi'(x)\| = 1$ B. $\|\phi'(x)\| \to \infty$ C. $\|\phi'(x)\| > 1$ D. $\|\phi'(x)\| < 1$	**2**	**CO1**
Q1 (v)	Which of the following two methods are similar at initial level for finding the roots of algebraic and transcendental equations: A. Bisection Method and Regula-Falsi Method B. Fixed Point Iteration Method and Secant Method C. Newton Raphson Method and Regula-Falsi Method D. Secant Method and Regula-Falsi Method	**2**	**CO1**
Q1 (vi)	Secant method for finding the root of the algebraic and transcendental equations is: A. $x_{k+1} = x_k + \frac{(x_k - x_{k-1})}{f_k - f_{k-1}} f_k, \quad k = 1,2,3, \ldots\ldots\ldots$ B. $x_{k+1} = x_k - \frac{(x_k - x_{k-1})}{f_k - f_{k-1}} f_k, \quad k = 1,2,3, \ldots\ldots\ldots$ C. $x_{k+1} = x_k - \frac{(x_k + x_{k-1})}{f_k - f_{k-1}} f_k, \quad k = 1,2,3, \ldots\ldots\ldots$ D. $x_{k+1} = x_k + \frac{(x_k - x_{k-1})}{f_k + f_{k-1}} f_k, \quad k = 1,2,3, \ldots\ldots\ldots$	**2**	**CO1**
Q1 (vii)	What is the approximate range of u in Gauss's forward formula? A. $-0.5 < u < 0$ B. $-0.25 < u < 0.25$ C. $0 < u < 0.5$ D. $0 < u < 1$	**2**	**CO2**
Q1 (viii)	The relation between divided difference and ordinary difference is A. $\frac{\Delta y_0}{x_1} = \frac{y_0 - y_1}{x_0 - x_1} = \frac{\Delta y_0}{h}$ B. $\frac{\Delta y_0}{x_1} = \frac{y_0 - y_1}{x_0 - x_1} = \Delta y_0$ C. $\frac{\Delta y_0}{x_1} = \frac{y_0 - y_1}{x_0 - x_1} = h\Delta y_0$ D. None of these	**2**	**CO2**

Q1 (ix)	Shifting the origin in Gauss's backward formula one have A. Stirling Formula B. Bessel's formula C. Newton's formula D. None of these	**2**	**CO2**
Q1 (x)	By Taylor's theorem, the series about a point $x = x_0$ is given by A. $y = y_0 + x_0(y')_0 + \frac{x_0^2}{2!}(y'')_0 + \frac{x_0^3}{3!}(y''')_0 + \dots\dots\dots$ B. $y = y_0 + (x - x_0)(y')_0 + \frac{(x-x_0)^2}{2!}(y'')_0 + \frac{(x-x_0)^3}{3!}(y''')_0 + \dots\dots\dots$ C. $y = y_0 + (x + x_0)(y')_0 + \frac{(x+x_0)^2}{2!}(y'')_0 + \frac{(x+x_0)^3}{3!}(y''')_0 + \dots\dots\dots$ D. None of these	**2**	**CO4**
Q1 (xi)	In the Gauss elimination method for solving a system of linear algebraic equations, triangularzation leads to A. Diagonal matrix B. Lower triangular matrix C. Upper triangular matrix D. Singular matrix	**2**	**CO4**
Q1 (xii)	The divided difference are : . A. Asymmetrical in all their arguments B. Symmetrical in all their arguments C. Inverse in all their arguments D. None of these	**2**	**CO2**
Q1 (xiii)	In Euler's method: Given initial value problem $\frac{dy}{dx} = f(x, y)$, with $y(x_0) = y_0$, then the approximation is given by A. $y_{n+1} = y_n + hf(x_{n-1}, y_{n-1})$ where $h = \frac{x_n - x_0}{n}$ B. $y_{n+1} = y_n + hf(x_n, y_n)$ where $h = \frac{x_n - x_0}{n}$ C. $y_{n+1} = y_n$ D. None of these	**2**	**CO4**

Q1 (xiv)	Given y_0, y_1, y_2, y_3 corresponding to x_0, x_1, x_2, x_3 for function $y = f(x)$. Let $f(x)$ is a polynomial of degree three. Then by Simpson's three eight rule, the integral $J = \int_a^b f(x)dx$ is equivalent to A. $J = \frac{3}{8}h[y_0 + 3y_1 + 3y_2 + y_3]$ B. $J = \frac{1}{3}h[y_0 + 4y_1 + y_2]$ C. $J = \frac{3}{8}h[y_0 + 4y_1 + 3y_2 + 2y_3]$ D. None of these	**2**	**CO3**
Q1 (xv)	Given initial value problem $\frac{dy}{dx} = f(x,y)$ where $y(x_0) = y_0$. In Runge-Kutta Method A. $y_{n+1} = y_n + \frac{1}{6}(k_1 + k_2 + k_3 + k_4)$ B. $y_{n+1} = y_n + \frac{1}{6}(k_1 + 4k_2 + 2k_3 + k_4)$ C. $y_{n+1} = y_n + \frac{3}{8}(k_1 + k_2 + k_3 + k_4)$ D. $y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$	**2**	**CO4**
Q1 (xvi)	Using fixed point iteration method $x_{n+1} = \phi(x_n)$, the equation $x^3 + x^2 - 1 = 0$ has been reduced into $x = \frac{1}{\sqrt{1+x}}$, then $\phi'(x)$ will be: A. $-\frac{1}{2(1+x)^{3/2}}$ B. $\frac{1}{2(1+x)^{3/2}}$ C. $-\frac{1}{(1+x)^{3/2}}$ D. $\frac{1}{(1+x)^{3/2}}$	**3**	**CO1**
Q1 (xvii)	Which of the following expression is true for applying Newton-Raphson method to obtain the approximate value of $(17)^{\frac{1}{3}}$: A. $x_{k+1} = \frac{2x_k^3+17}{3x_k^2}, \quad k = 0,1,2 \dots \dots \dots$ B. $x_{k+1} = \frac{2x_k^3-17}{3x_k^2}, \quad k = 0,1,2 \dots \dots \dots$ C. $x_{k+1} = \frac{x_k^3-17}{3x_k^2}, \quad k = 0,1,2 \dots \dots \dots$ D. $x_{k+1} = \frac{x_k^3+17}{3x_k^2}, \quad k = 0,1,2 \dots \dots \dots$	**3**	**CO1**

<table>
<tr>
<td>Q1 (xviii)</td>
<td>Let $f(x) = a^2x^n + bx^{n-1} + cx^{n-2} + \dots\dots\dots + kx + l$ and $\Delta f(x) = f(x+h) - f(x)$, then $\Delta^n f(x)$ will be:

A. $a\, n!\ h^n$
B. $a^2\, n!\ h^n$
C. $n!\ h^n$
D. $a\, n!$</td>
<td>3</td>
<td>CO2</td>
</tr>
<tr>
<td>Q1 (xix)</td>
<td>The third order backward difference $\nabla^3 y_3$ can be expressed as:

A. $y_3 - 3y_2 + 3y_1 - y_0$
B. $y_3 + 3y_2 + 3y_1 + y_0$
C. $y_3 + 3y_2 - 3y_1 - y_0$
D. $-y_3 - 3y_2 + 3y_1 + y_0$</td>
<td>3</td>
<td>CO2</td>
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<tr>
<td>Q1 (xx)</td>
<td>The polynomial $f(x) = 2x^3 - 3x^2 + 3x - 10$ can be written in factorial notation as:

A. $f(x) = 2[x]^3 - 3[x]^2 - 2[x] - 10$
B. $f(x) = 2[x]^3 + 3[x]^2 + 2[x] - 10$
C. $f(x) = 2[x]^3 + 2[x]^2 + 3[x] + 10$
D. $f(x) = 2[x]^3 + 3[x]^2 - 2[x] - 10$</td>
<td>3</td>
<td>CO2</td>
</tr>
<tr>
<td>Q1 (xxi)</td>
<td>The following values are used to find a polynomial of degree four using Gauss's Forward formula:
<table>
<tr><td>x:</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>$f(x)$:</td><td>1</td><td>-1</td><td>1</td><td>-1</td><td>1</td></tr>
</table>
Then the value of u will be:

A. $u = x - 3$
B. $u = x + 3$
C. $u = x - 1$
D. $u = x + 1$</td>
<td>3</td>
<td>CO2</td>
</tr>
<tr>
<td>Q1 (xxii)</td>
<td>The third divided difference with arguments 2, 4, 9, 10 of the function $f(x) = x^3 - 2x$ will be:

A. 1
B. 2
C. 3
D. 4</td>
<td>3</td>
<td>CO2</td>
</tr>
</table>

Q1 (xxiii)	In LU decomposition , the equations $3x+2y+7z=4; 2x+3y+z=5; 3x+4y+z=7$ can be represented as: $\begin{bmatrix}1&0&0\\l_{21}&1&0\\l_{31}&l_{32}&1\end{bmatrix}\begin{bmatrix}u_{11}&u_{12}&u_{13}\\0&u_{22}&u_{23}\\0&0&u_{33}\end{bmatrix}=\begin{bmatrix}3&2&7\\2&3&1\\3&4&1\end{bmatrix}$ Then the value of l_{32} is: A. $-\frac{11}{3}$ B. $-\frac{8}{5}$ C. $\frac{5}{3}$ D. $\frac{6}{5}$	**3**	**CO4**
Q1 (xxiv)	**Match the following:** A. Newton-Raphson — 1. Integration B. Runge-kutta — 2. Root finding C. Gauss-seidel — 3. Ordinary Differential Equations D. Simpson's Rule — 4. Solution of system of Linear Equations A. A2-B3-C4-D1 B. A3-B2-C1-D4 C. A1-B4-C2-D3 D. A4-B1-C2-D3	**3**	**CO3**
Q1 (xxv)	In Newton-Cotes formula, if $f(x)$ is interpolated at equally spaced nodes by a polynomial of degree two then it represents A. Trapezoidal rule B. Simpson's one third rule C. Simpson's three eight rule D. None of these	**3**	**CO3**

PART B

The link for PART B will be available from 10:00 AM on 6th July 2020 to 10:00 AM on 7th July 2020. Solve the problems in PART B on a plain A4 sheets and write your name, roll number and SAP ID on each page and then scan them into a single PDF file. Name the file as SAP ID _BRANCH NAME_ROLL NUMBER (for example: 500077624_CCVT_ R103219023.pdf) and upload that PDF file through the link provided over there. PART B solutions sent through WhatsApp or email will not be entertained.

Q2	Let $f(x)$ be a non-zero function such that n^{th} derivative of it is equal to the function itself. Find the smallest positive root of the equation $x-\frac{1}{f(x)}=0$ by using method of False position, correct to three decimal places. **OR**	**8**	**CO1**

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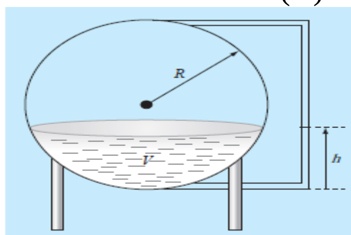
<td>You are designing a spherical tank to hold water (See Fig below). The volume of liquid it can hold, can be computed by $V = \frac{\pi h^2[3R-h]}{3}$. If the radius R=3 m, what depth (h) must the tank be filled to so that it holds a volume (V) of 30 m^3?


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<td></td>
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<tr>
<td>Q3</td>
<td>Using Newton Backward formula, estimate the population for the year 1925 from the following table:
<table>
<tr><td>Year (x)</td><td>1891</td><td>1901</td><td>1911</td><td>1921</td><td>1931</td></tr>
<tr><td>Population (y) (in thousands)</td><td>46</td><td>66</td><td>81</td><td>93</td><td>101</td></tr>
</table>
</td>
<td>8</td>
<td>CO2</td>
</tr>
<tr>
<td>Q4</td>
<td>Use Simpson’s rule dividing the range into ten equal parts to show that

$$\int_0^1 \frac{\ln(1+x^2)}{1+x^2}\,dx = 0.173$$
</td>
<td>8</td>
<td>CO3</td>
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<tr>
<td>Q5 (A)</td>
<td>Solve equations $27x + 6y - z = 85$; $x + y + 54z = 110$; $6x + 15y + 2z = 72$ using Gauss-Seidel method. Use only four iterations.</td>
<td>8</td>
<td>CO4</td>
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<tr>
<td>Q5 (B)</td>
<td>Using Runge-Kutta method of fourth order, solve for y at $x = 0.2$ from $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$ given $y(0) = 1$ (take $h = 0.2$).</td>
<td>8</td>
<td>CO4</td>
</tr>
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